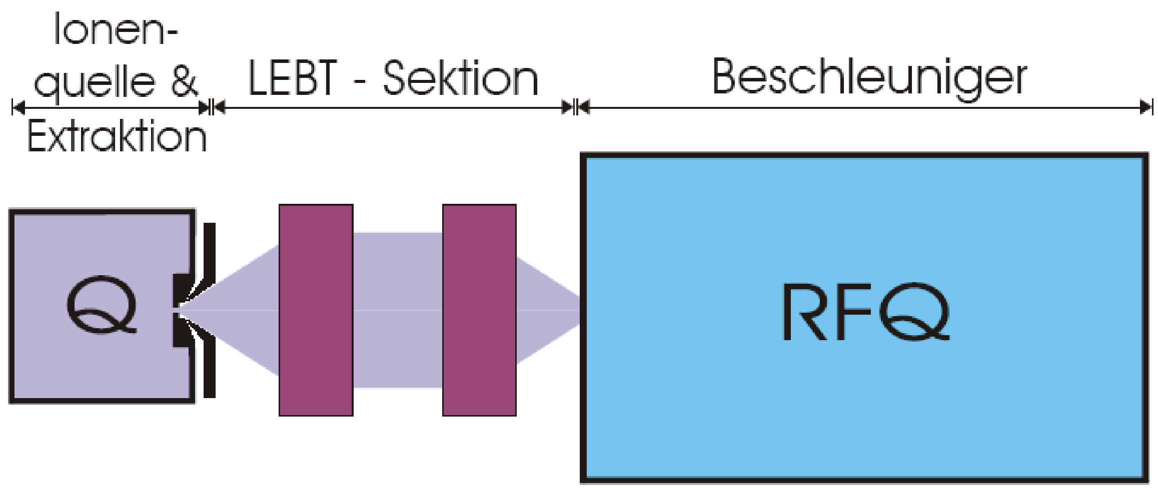
14) First acceleration stage: RFQ accelerators

We did discuss the ion production in ion sources, the beam formation and the beam transport towards the first acceleration stage. The first acceleration stage in a modern heavy ion accelerator is the Radio Frequency Quadrupole (RFQ) accelerator.

The typical set-up of a Low Energy Beam Transport (LEBT) is shown here. The beam transport consist on lenses, that can be electrostatic or magnetic. In order to keep the beam round (same emittance in both directions) cylinder symmetric lenses are used commonly.



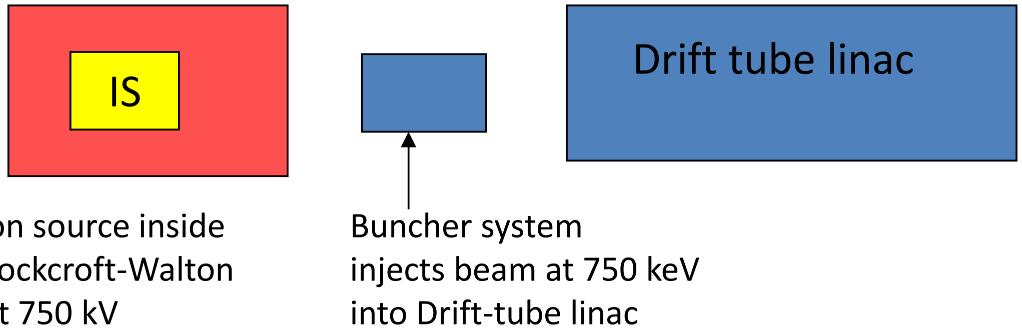
The RFQ accelerator is a linac of a relatively recent date. It was invented in the late 60's by Kapchinsky of the Institute of Experimental and Theoretical Physics (ITEP) in Moscow. Teplyakov at the same institute constructed the cavity. The cavity is excited in an electric quadrupole mode in which the RF electric field is concentrated near the vane tips and produces a transverse RF quadrupole focusing force on the beam.

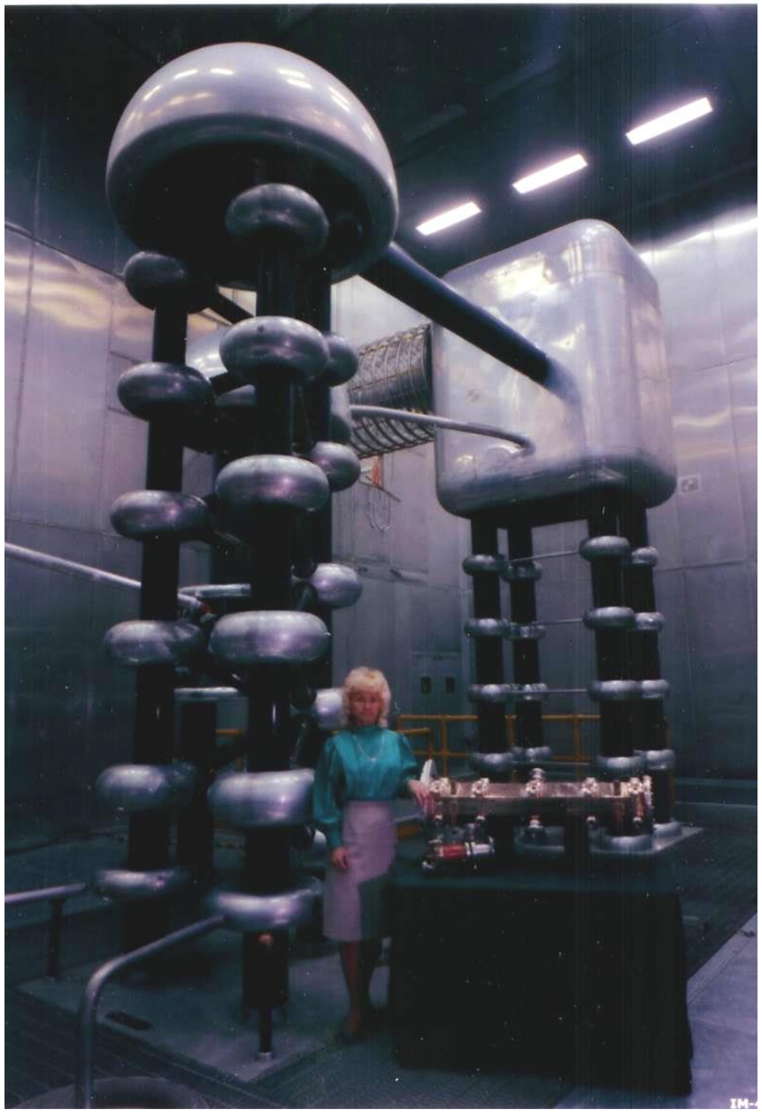
Important contributions to the field have been made by the Los Alamos National Laboratory (LANL) in the 70's and 80's.

By then the RFQ became the standard injector for ion accelerators, replacing electrostatic accelerators like the Cockcroft-Walton.

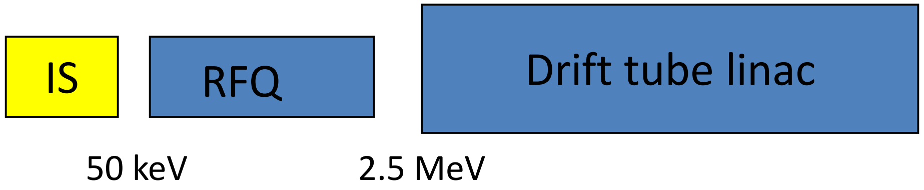
1-MeV “Bear” RFQ which is the first and only accelerator that has operated in space is shown with Cockcroft-Walton that it could replace.

Pre-RFQ linac architecture used an ion source in a large Cockcroft-Walton, an inefficient buncher (poor capture efficiency) and weak magnetic focusing at low velocities, resulting in poor beam quality which typically led to beam loss.

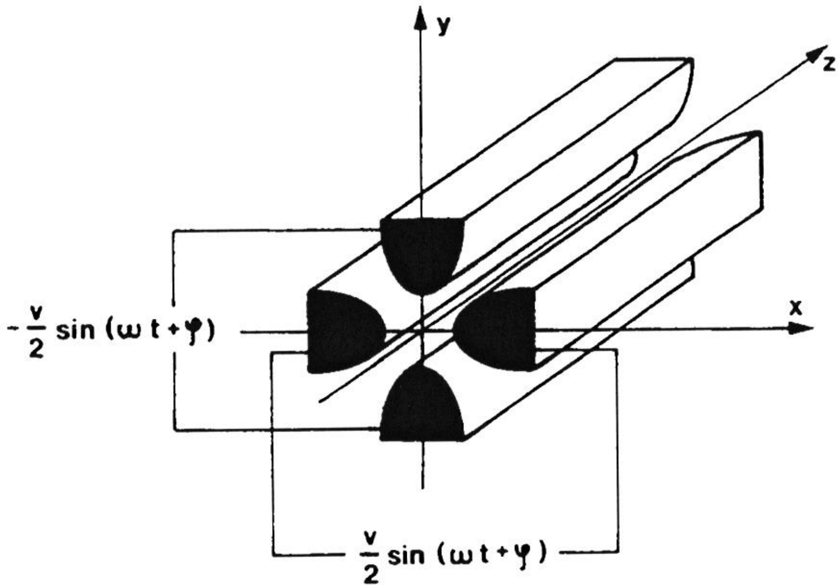




RFQ provides stronger (electric) focusing,
lowers injection velocity and eliminates the large Cockcroft-Walton. The RFQ raises injection energy into the DTL (stronger magnetic focusing. As option it eliminates an external buncher system. The RFQ leads to beam quality improvements.



The salient features of the RFQ are that it bunches focuses and accelerates charged particles by using RF-fields only. The RFQ is very well suited to accelerate ions in the low energy range up to 1-2 MeV/u. For protons up to 3-5 MeV.



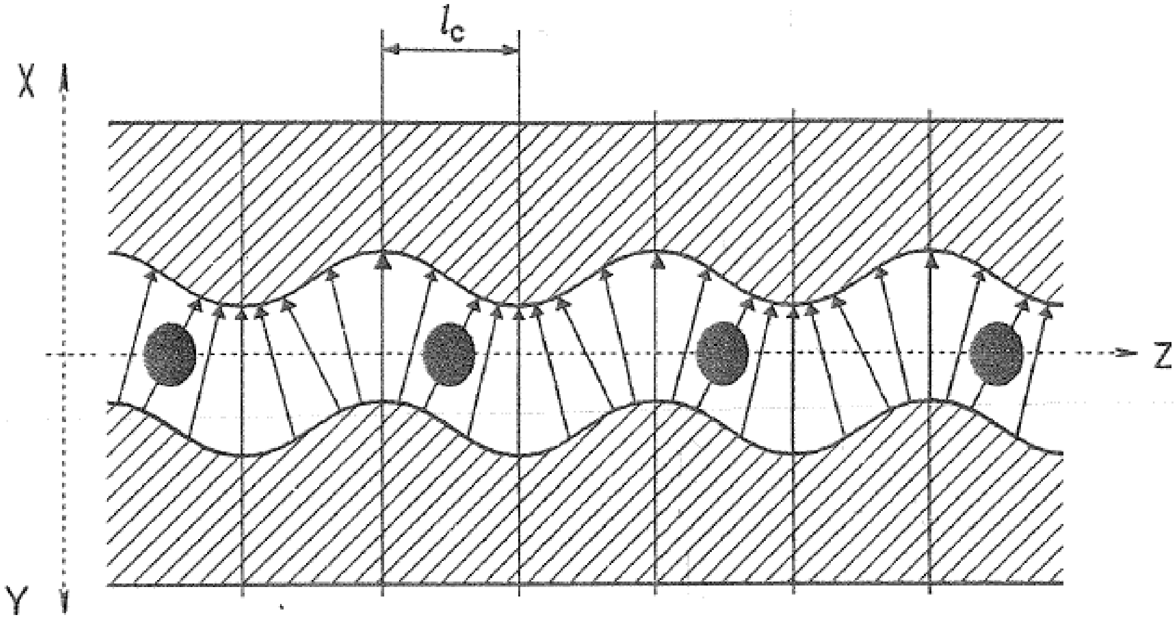
The RFQ is an electric quadrupole having alternating voltages on its electrodes. The particles move in z-direction and undergo alternating gradient focusing (AG-focusing), due to the time varying fields.

Instead of the usual sequence of 'spatially' distributed quadrupoles with alternating polarities, the same effect can be produced by a long, single quad with time-varying fields.

For some RF cavity problems involving time-dependent fields, the fields can be obtained from a time-dependent scalar potential that satisfies Laplace's equation. This solution is called the quasistatic approximation and is valid when the field variations are determined by cavity structural elements whose geometrical sizes are small compared with the RF wavelength. In particular the quasistatic approximation applies near the beam aperture when the beam aperture is small compared with the wavelength

The longitudinal fields can be created by perturbing the electrodes of the a.c. quadrupole in a periodic way. A part of the originally transverse field is deviated into the longitudinal direction.

The shape of the perturbation will come out from the field calculations.
The period of the perturbation is such as to create synchronism between modulation and particle, necessary for acceleration.



14.1) Electric field established by the RFQ-electrodes

We will first investigate the fields relevant for the beam dynamics and later we will look to the distinct problem of the cavities. We are using now the quasistatic approximation to calculate the fields between the electrodes by solving Laplace's equation. The Laplace equation in cylindrical coordinates is

$$\Delta U(r,\theta,z) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{\partial^2 U}{\partial z^2} = 0 \qquad (14.1)$$

with the time dependence $\phi(r,\theta,z,t) = U(r,\theta,z)\sin(\omega t + \varphi)$

To evaluate the solution of (14.1) we make a separation Ansatz

$$U(r,\theta,z) = R(r)\cdot Q(\theta)\cdot Z(z) \qquad (*)$$

$$Q(\theta)\cdot Z(z)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + R(r)\cdot Z(z)\frac{1}{r^2}\frac{\partial^2 Q}{\partial\theta^2} + R(r)\cdot Q(\theta)\frac{\partial^2 Z}{\partial z^2} = 0$$

We divide the equation by $R(r)\cdot Q(\theta)\cdot Z(z)$ and get

$$\frac{1}{R(r)}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \frac{1}{Q(\theta)}\frac{1}{r^2}\frac{\partial^2 Q}{\partial\theta^2} = -\frac{1}{Z(z)}\frac{\partial^2 Z}{\partial z^2} = f(\theta,r) = const = k_z^2$$

$$\Rightarrow \quad \frac{\partial^2 Z}{\partial z^2} + k_z^2 Z(z) = 0 \qquad (14.2)$$

The complete solution is $Z(z) = \sum_m \left[a_m \cos(k_z z) + b_m \sin(k_z z)\right]$ (14.2b)

The same for Q: $\frac{\partial^2 Q}{\partial\theta^2} + k_\theta^2 Q(\theta) = 0$ (14.3)

The complete solution is $Q(\theta) = \sum_n \left[c_n \cos(k_\theta z) + d_n \sin(k_\theta z)\right]$ (14.3b)

We get with the constants the radial equation

$$\frac{1}{R(r)}\frac{1}{r}\frac{\partial R}{\partial r}+\frac{1}{R(r)}\frac{\partial^2 R}{\partial r^2}-\left(k_z^2+\frac{k_\theta^2}{r^2}\right)=0 \qquad (14.4)$$

In case of $k_z = 0$ we derive from (14.4)

$$R(r)=A_{k\theta}r^{k\theta} \qquad (14.4b)$$

In case of $k_z \neq 0$ we derive from (14.4)

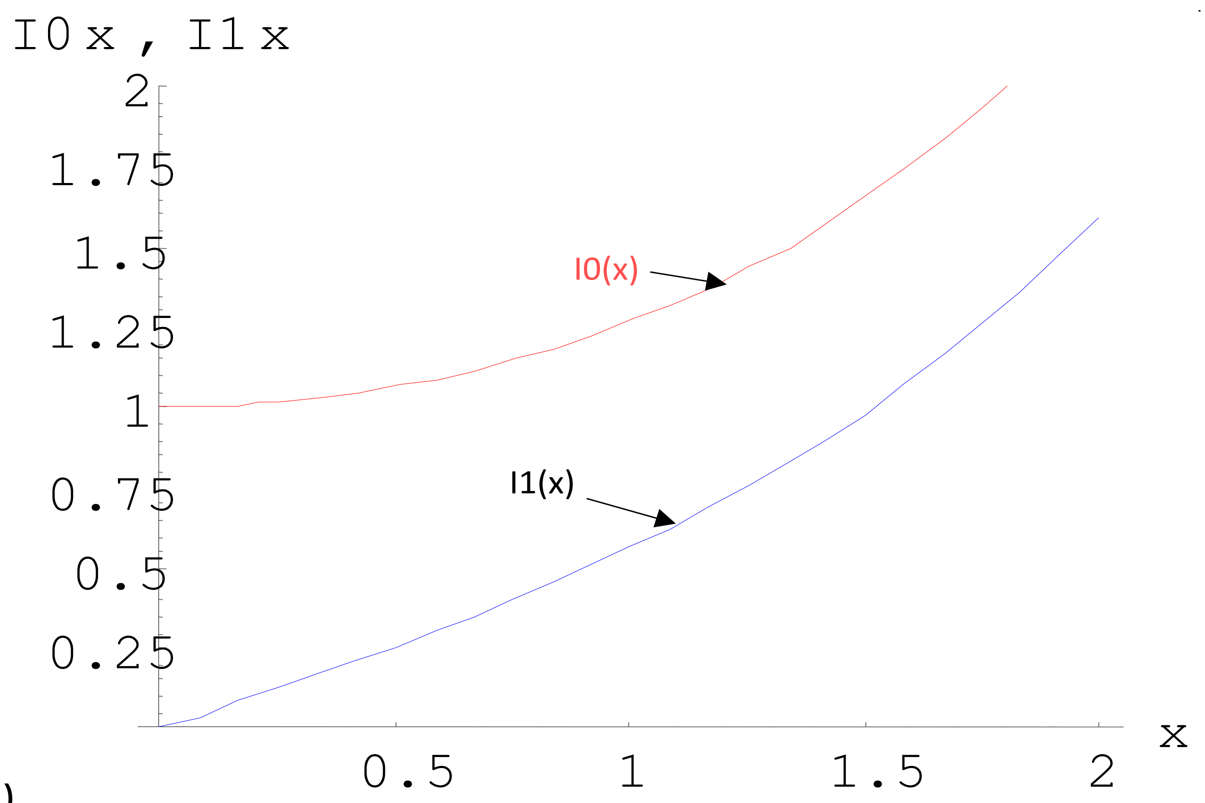
$$R(r)=A_{k\theta}I_0(k_z r) \qquad (14.4c)$$

$I_{k\theta}$ is the modified Bessel function.

Some basic boundary conditions in case of the RFQ have to be taken into account:

$$U(r,\theta,-z)=U(r,\theta,z) \qquad (14.5a)$$

Modified Bessel Functions $I_0(x)$ and $I_1(x)$



$I_0(x) \cong 1 + x^2/4 + \ldots,$ $\qquad I_1(x) \cong x/2 + \ldots.$

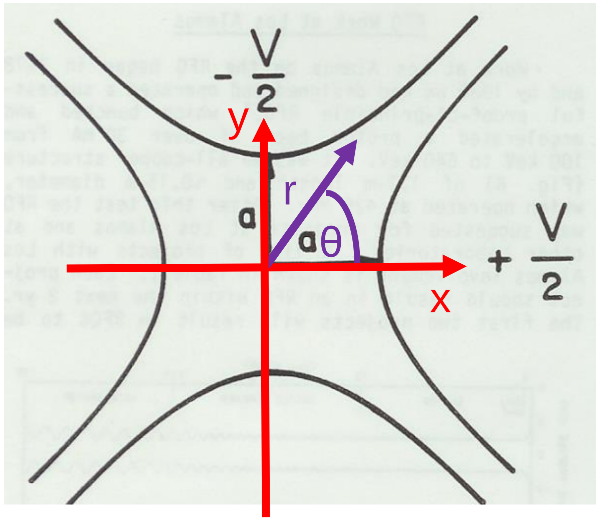
$z = 0$ is always a maximum or a minimum.

$$U(r,\theta,z) = U(r,\theta,z+\beta\lambda) \qquad (14.5b)$$

$$U(r,-\theta,z) = U(r,\theta,z) \qquad (14.5c)$$

$$U(r,\pi\pm\theta,z) = U(r,\theta,z) \qquad (14.5d)$$

$$-U\left(r,\theta\pm\frac{\pi}{2},\frac{\beta\lambda}{2}-z\right) = U(r,\theta,z) \qquad (14.5e)$$



From (14.5a) and (14.2b) follows $b_m = 0\ \forall m$ and $Z(z) = \sum_m a_m \cos(k_z z)$ (14.2c)

From (14.5b) and (14.2c) we get $\sum_m a_m \cos(k_z z) = \sum_m a_m \cos(k_z[z+\beta\lambda])$

$$\Rightarrow \quad k_z\beta\lambda = 2\pi\cdot m \quad \Rightarrow \quad k_z = m\frac{2\pi}{\beta\lambda}$$ with m = 0,1,2… (14.6)

$$Z(z) = \sum_m a_m \cos\left(\frac{2\pi\cdot m}{\beta\lambda}z\right) \qquad (14.2d)$$

From (14.5c) and (14.3b) follows $d_n = 0 \ \forall n$ and $Q(\theta) = \sum_n c_n \cos(k_\theta \theta)$ (14.3c)

From (14.5d) and (14.3c) we get analogue to the calculation above

$$Q(\theta) = \sum_n c_n \cos(2n\theta) \qquad (14.3d)$$

Finally we get for the potential $U(r,\theta,z)$ and $k = 2\pi/\beta\lambda$

$$U(r,\theta,z) = \sum_{n,m>0} A_{mn} \cos(mkz)\cos(2n\theta) I_{2n}(mkr) + \sum_n A_{0n} r^{2n} \cos(2n\theta) \qquad (14.7)$$

Using (14.5e) we can relate m and n with

$$\sum_{n,m>0} A_{mn} \cos(mkz)\cos(2n\theta) I_{2n}(mkr) + \sum_n A_{0n} r^{2n} \cos(2n\theta) =$$

$$-\sum_{n,m>0} A_{mn} \cos\left(mk\left(\frac{\beta\lambda}{2} - z\right)\right)\cos\left(2n\left(\theta \pm \frac{\pi}{2}\right)\right) I_{2n}(mkr) + \sum_n A_{0n} r^{2n} \cos\left(2n\left(\theta \pm \frac{\pi}{2}\right)\right)$$

$$\Rightarrow \quad m + n = 2p + 1 \quad , \quad p = 1,2,3\ldots$$

If m = 0 we get n = 2p+1and therefore the second term of (14.7) is also satisfied. If m is even, n must be odd and vice versa. Thus we finally get

$$U(r,\theta,z)=\sum_{n,m>0}A_{mn}\cos(mkz)\cos(2n\theta)I_{2n}(mkr)+\sum_{n}A_{0n}r^{2n}\cos(2n\theta)$$

$$=\frac{U_0}{2}\left(\sum_{n,m>0}\tilde{A}_{mn}\cos(mkz)\cos(2n\theta)I_{2n}(mkr)+\sum_{n}\tilde{A}_{0n}r^{2n}\cos(2n\theta)\right) \qquad (14.7b)$$

$$\text{with}\quad k=\frac{2\pi}{\beta\lambda}\quad\text{and}\quad m+n=2p+1,\ p=1,2,3...$$

The lowest order of (14.7b) is p = 0 → m = 0; n = 1 or m = 1; n = 0

$$U(r,\theta,z)=\frac{U_0}{2}\left(\tilde{A}_{10}\cos(kz)I_0(kr)+\tilde{A}_{01}r^2\cos(2\theta)\right) \qquad (14.8)$$

If the rod potential is $U_0/2$. Usually the lowest order terms are sufficient to describe the RFQ fields. This is called the two-term potential function description and is starting point for RFQ design. For this we can derive the constants from the boundary conditions. In (14.8) the second term describes the quadrupole focusing and the first term the acceleration. At the rod surface we get

$$U(a,0,0)=\frac{U_0}{2}=\frac{U_0}{2}\left(\tilde{A}_{10}I_0(ka)+\tilde{A}_{01}a^2\right)\ \Rightarrow\ \tilde{A}_{01}a^2=\chi=1-\tilde{A}_{10}I_0(ka)$$

$$U(ma,0,\frac{\beta\lambda}{2}) = \frac{U_0}{2} = \frac{U_0}{2}\left(\tilde{A}_{10} I_0(kma)\cos(k\frac{\beta\lambda}{2}) + \tilde{A}_{01} m^2 a^2\right) \quad \Rightarrow \quad \chi \cdot m^2 = 1 + \tilde{A}_{10} I_0(kma)$$

Thus we get $\chi \cdot m^2 = \left(1 + \tilde{A}_{10} I_0(ka)\right) \cdot m^2 = 1 + \tilde{A}_{10} I_0(kma)$

$$\Rightarrow \quad m^2 - 1 = \tilde{A}_{10}\left(I_0(ka) \cdot m^2 + I_0(kma)\right)$$

$$\Rightarrow \quad \tilde{A}_{10} = \frac{m^2 - 1}{I_0(ka) \cdot m^2 + I_0(kma)} \qquad (14.9)$$

$$\Rightarrow \quad \tilde{A}_{01} = \frac{1 - \tilde{A}_{10} I_0(ka)}{a^2} = \frac{1}{a^2}\left(1 - \frac{(m^2 - 1) I_0(ka)}{I_0(ka) \cdot m^2 + I_0(kma)}\right)$$

$$\Rightarrow \quad \tilde{A}_{01} = \frac{1}{a^2}\left(\frac{I_0(ka) + I_0(kma)}{I_0(ka) \cdot m^2 + I_0(kma)}\right) = \frac{\chi}{a^2} \qquad (14.10)$$

The intervane voltage divides into $\chi = 1 + \tilde{A}_{10} I_0(ka) \quad \Rightarrow \quad U_0 = U_0\chi + U_0 \tilde{A}_{10} I_0(ka)$

The intervane voltage is composed of a part for focusing ($U_0\chi$) and another part for acceleration ($U_0 \tilde{A}_{10} I_0(ka)$).

In case of m=1 (no modulation) we get $\tilde{A}_{10} = 0\,, \chi = 1$

→ no acceleration only focusing, $\tilde{A}_{10} = \text{acceleration efficiency}\ , \chi = \text{focusing efficiency}$

From (14.8) we can calculate the electric field established by the RFQ electrodes:

$$E_r = -\frac{\partial U}{\partial r} = -\frac{U_0}{2}\left(\tilde{A}_{10} k \cos(kz) I_1(kr) + 2\tilde{A}_{01} r \cos(2\theta)\right), \quad \frac{\partial I_0(u)}{\partial u} = I_1(u) \qquad (14.11a)$$

$$E_\theta = -\frac{1}{r}\frac{\partial U}{\partial \theta} = U_0 \tilde{A}_{01} r \sin(2\theta) \qquad (14.11b)$$

$$E_z = -\frac{\partial U}{\partial z} = \frac{U_0}{2}\tilde{A}_{10} k \sin(kz) I_0(kr) \qquad (14.11c)$$

In case of m=1 the longitudinal field is 0 (no acceleration) and the transverse field correspond to a pure quadrupole field.

$$E_r = U_0\left[-\frac{r}{a^2}\cos(2\theta) + \frac{\tilde{A}_{01} r}{a^2} I_0(ka)\cos(2\theta) - \frac{\tilde{A}_{10} k}{2} I_1(kr)\cos(kz)\right]$$

focusing term — acc. term (reduces the focusing strength) — rf-defocusing

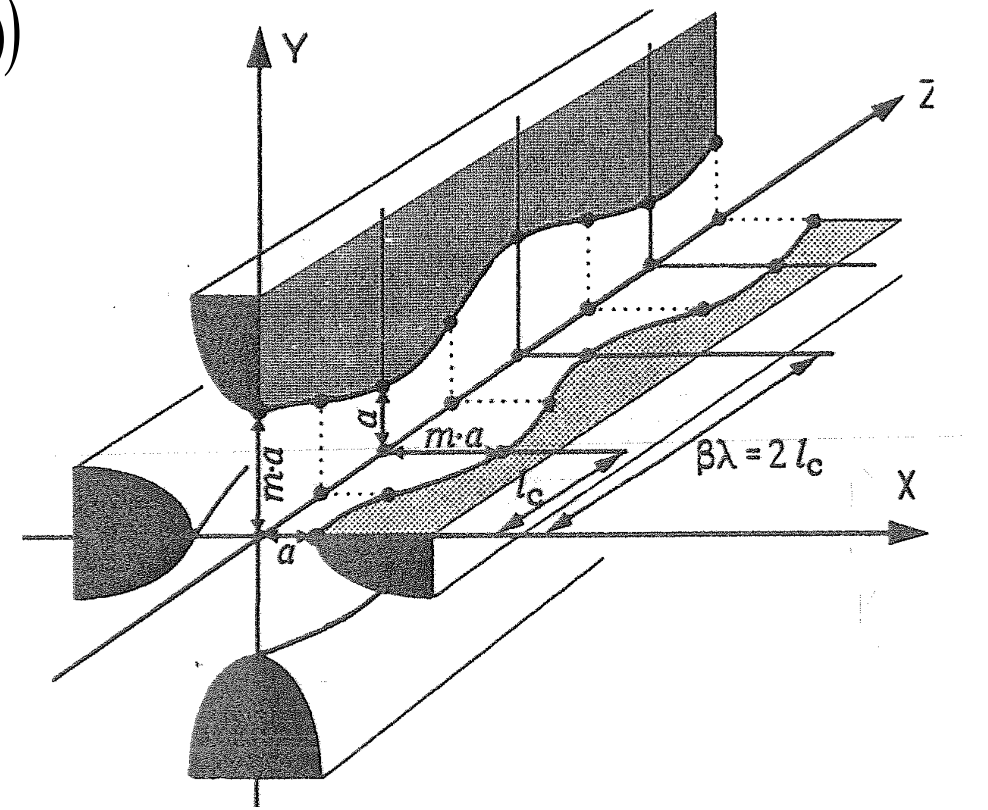
The vane electrode surface is defined by (14.8):

$$\pm\frac{U_0}{2}=\frac{U_0}{2}\left(\tilde{A}_{10}\cos(kz)I_0(kr)+\tilde{A}_{01}r^2\cos(2\theta)\right)$$

$$\pm 1=\tilde{A}_{10}\cos(kz)I_0(kr)+\chi\frac{r^2}{a^2}\cos(2\theta)$$

→ $$r^2\cos(2\theta)=\frac{a^2}{\chi}\left[\pm 1-\tilde{A}_{10}I_0(kr)\cos(kz)\right]$$

(14.12)



To produce the axial field, we sinusoidally modulate the vane tips along the axial direction.

This provides the sinusoidal on-axis electric field.

14.2) RFQ-cavities

The frequency choice determines the type of
the RFQ-resonator structure.

The 4-vane structure (> 200 MHz) and the
4-rod structure (< 200 MHz) are the most
common resonators.

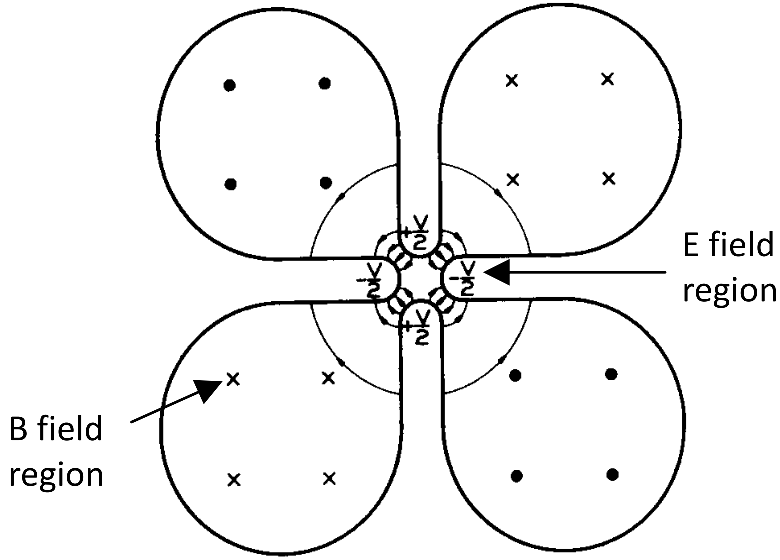
The 4-vane structure has been introduced by Los Alamos
National Laboratory and is a cavity.

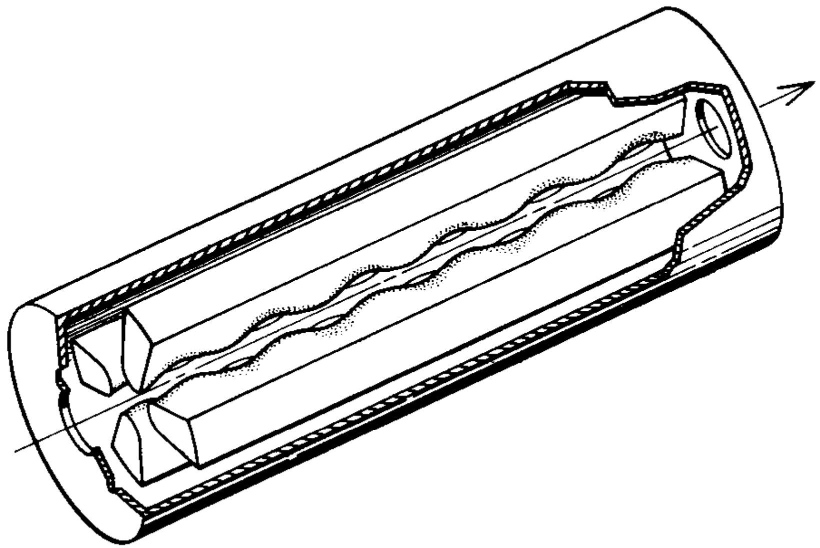
The 4-vane is exited in a transverse electric mode
the TE210 mode.

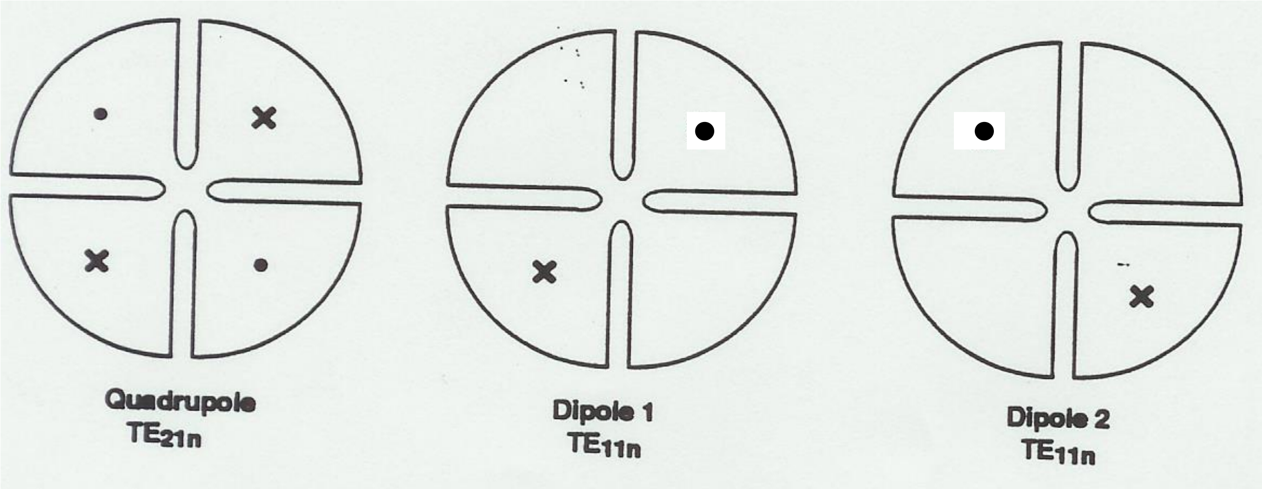
The transverse electric field is localized near vane tips.

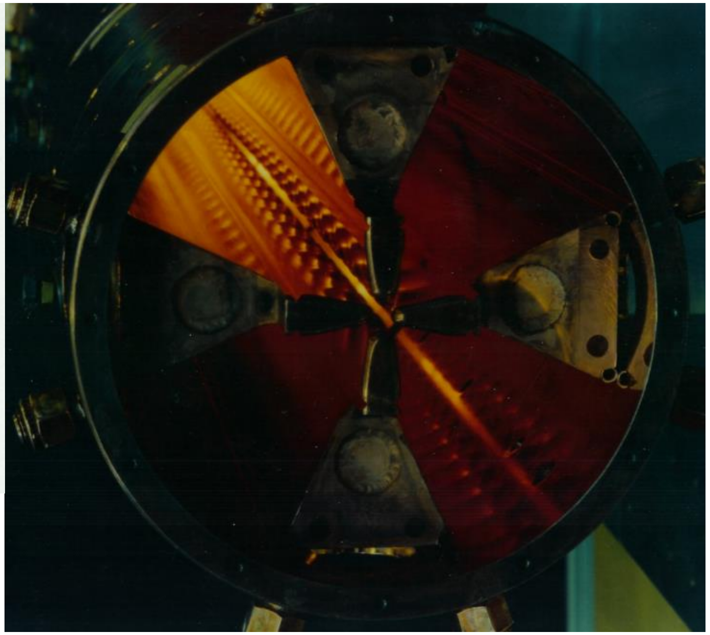
The magnetic field is longitudinal localized in the four
outer quadrants. The efficiency is high, because the vane
charging currents are uniform along the vanes.

In case of the 4-vane resonator the suppression of
the dipole mode is required.









The operating mode will be mostly an admixture of these three modes if they are all close in frequency. The 4-vane RFQ must be designed and tuned to minimize the contributions of the two dipole modes.

The 4-rod RFQ has been developed at Frankfurt University by Prof. Alwin Schempp. This RFQ resembles a transmission line. The vacuum tank does not really affect the resonance frequency.

The 4-rods are charged via the stems. Each stem connects to an opposite pair of rods.
→ opposite rods are shorted out

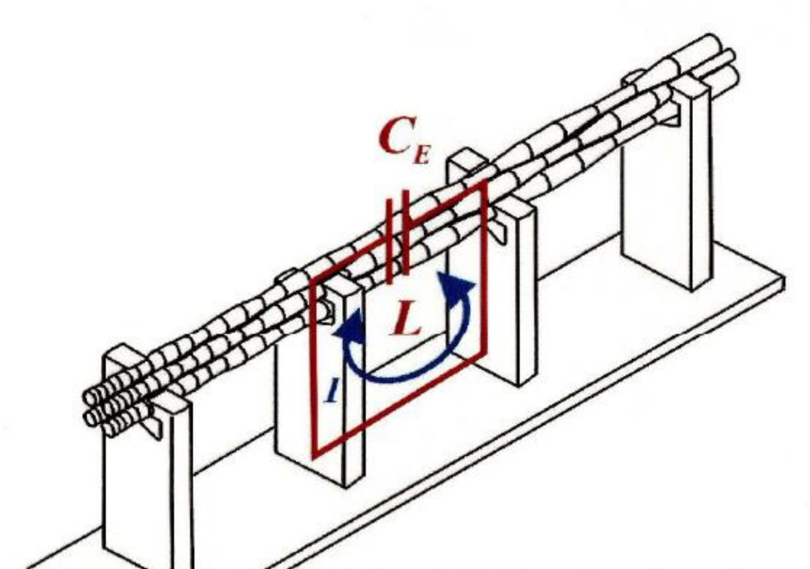
The rods are presently short vanes cut on a milling machine, no circular rods cut on a lathe.

→ Very compact, easy to tune and an excellent dipole mode suppression.

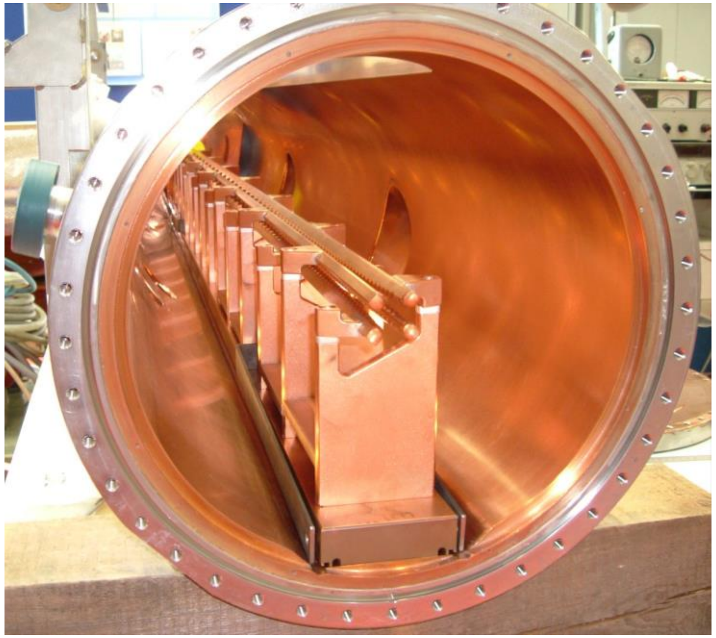
The rods can be cooled. Important is the peak surface electric field that can cause rf-electric break down.

About 50 years ago W.D. Kilpatrick analyzed data on rf-break down. The results were expressed by T.J. Boyd

$$f\,(\mathrm{MHz}) = 1{,}64 \cdot E_k^2 \exp\left[-\frac{8.5}{E_k}\right]$$



4-Rod RFQ basic cell



E_k is called the Kilpatrick limit. The criterion is conservative by today's standard.

$$E_s = \text{peak surface field} = b \cdot E$$

Here b = bravery factor with 1.5 < b < 2