5) Electron guns and electron beams

Until the thirties of the last century, electron beams were only used in highly specialized installations and in accelerators. At the beginning of the forties, electron beams found their application in high-frequency technologies (Klystrons, drift tubes, tetrodes etc.). Electron beams are used in TVs, electron microscopes, for welding- and lithographic devices.

Current: μA to kA

Beam energy: 0.1 to 200 keV

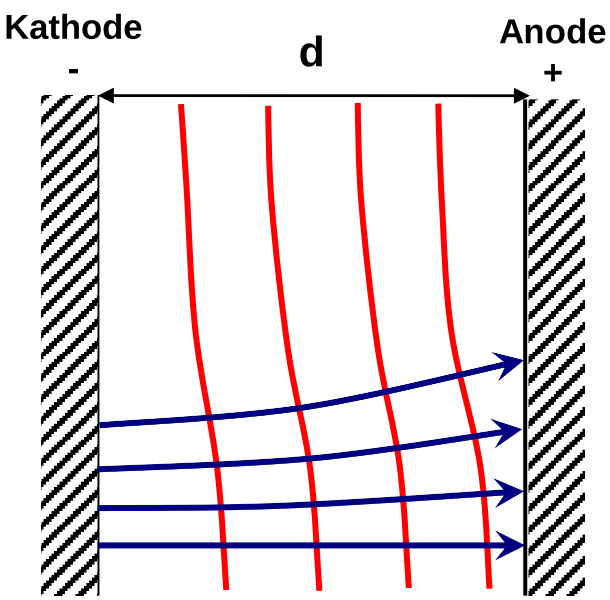
The influence of space charge has the be considered at high beam currents.

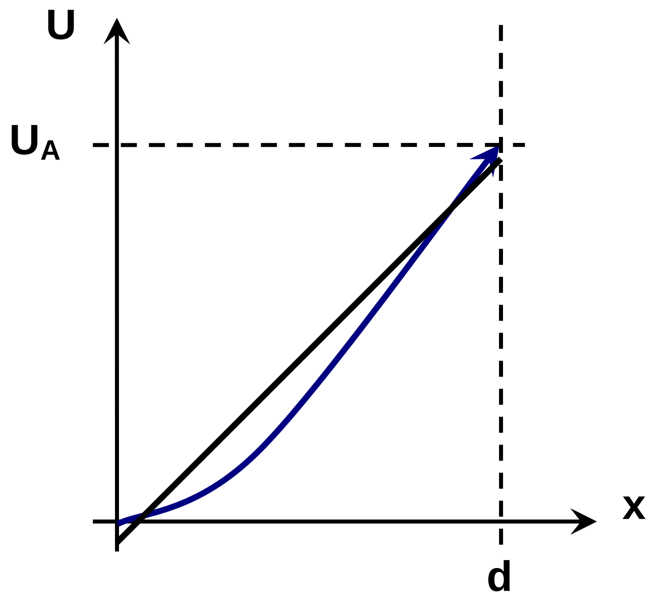
Simple diode system:

Deformation of the equipotential lines due to space charge

Space charge adjusts in that way, that the cathode does not see the anode voltage anymore
→ shielding

$$\left.\frac{dU}{dx}\right|_{x=0} = 0$$





Poisson equation: $$\Delta U = \frac{\partial^2 U}{\partial x^2} = -\frac{\rho}{\varepsilon_0} = \frac{j}{v \cdot \varepsilon_0} = \frac{j}{\varepsilon_0 \sqrt{\frac{2e}{m}}} \frac{1}{\sqrt{U}}$$

Ansatz: $U = A \cdot x^n$ => $U'' = A \cdot n(n-1) \cdot x^{n-2}$

➔ $n = 4/3$ and $A = \left(\frac{9}{4} \frac{j}{\varepsilon_0 \sqrt{\frac{2e}{m}}} \right)^{\frac{2}{3}}$ with boundary condition $U_A = A \cdot d^{4/3}$

therewith one gets the "spase charge limited region" for the emitted current:

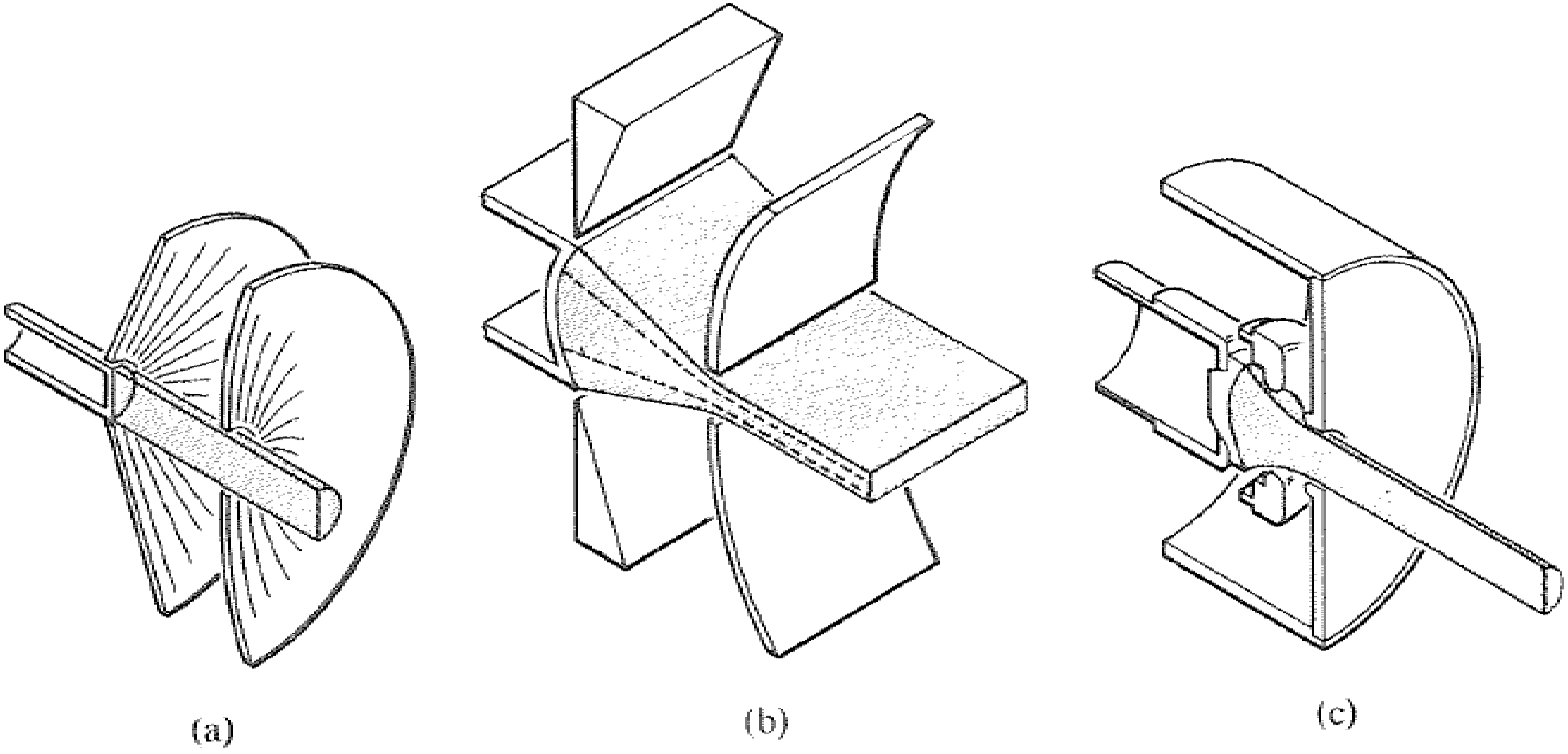
$$j = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \frac{U_A^{3/2}}{d^2}$$ → **Child-Langmuir-law** (5.1)

with $I = j * F$ F = emitting area

$$I = P \cdot U^{3/2}$$ with P = Perveance (5.2)

In the case of a planar diode system $$P = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \frac{F}{d^2}$$ (5.3)

The different geometries are shown below:



(a) Planar, (b) Cylindrical, generating a sheath-beam or (c) Spherical geometry.

In the case of a cylindrical or spherical symmetry we have the following expressions for the Poisson-equation:

Sphere: $$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)=-\frac{\rho}{\varepsilon_0}$$ with $$\rho=\frac{j}{v}=\frac{I}{4\pi\cdot r^2 v}$$ (5.4)

Cylinder: $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dU}{dr}\right)=-\frac{\rho}{\varepsilon_0}$$ with $$\rho=\frac{j}{v}=\frac{I}{2\pi r\cdot h\cdot v}$$ where h= length of the cylinder

Solutions from Langmuir and Blodgett

$$I=\frac{16}{9}\pi\cdot\varepsilon_0\sqrt{\frac{2e}{m}}\frac{U_A^{3/2}}{\alpha^2}$$ → **spherical symmetry** (5.5)

$$I=\frac{8}{9}\pi\cdot\varepsilon_0\sqrt{\frac{2e}{m}}\frac{U_A^{3/2}}{\beta^2 r}$$ → **cylindrical symmetry**

If we put (5.5) into (5.4) we get a differential equation for α.

$$6r\alpha\frac{d\alpha}{dr}+r^2\left(\frac{d\alpha}{dr}\right)^2+3r^2\alpha\frac{d^2\alpha}{dr^2}-1=0$$ (5.6)

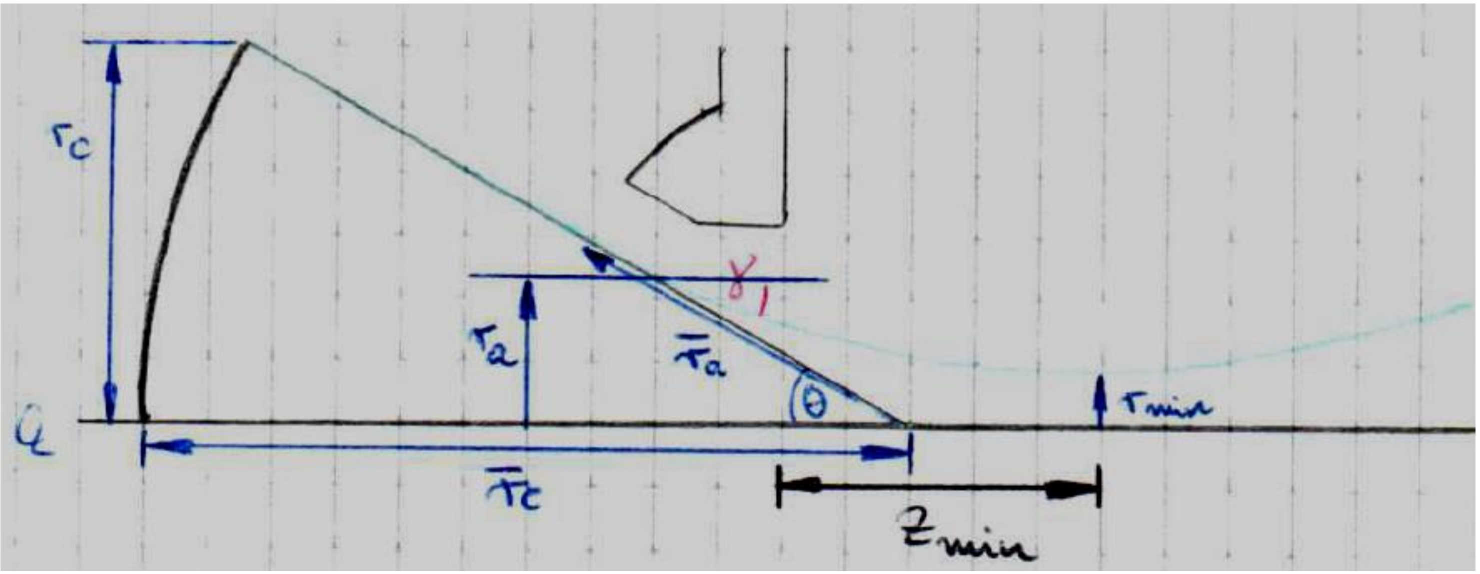
Normalized on the cathode radius r_c we get with $\gamma = \ln\left(\frac{r}{r_c}\right)$ from (5.6)

$$6r\alpha\frac{d\alpha}{dr} + r^2\left(\frac{d\alpha}{dr}\right)^2 + 3r^2\alpha\frac{d^2\alpha}{dr^2} - 1 = 0 \qquad (5.7)$$

is the cathode radius and $\alpha(r)$ can be represented as a series expansion.

$$\alpha = \gamma - \frac{3}{10}\gamma^2 + \frac{3}{40}\gamma^3 - \frac{63}{4400}\gamma^4 + \frac{13311}{6160000}\gamma^5 - \frac{27\cdot 10391}{10472\cdot 10^5}\gamma^6 \quad \text{with} \quad \gamma = \ln\left(\frac{r}{r_c}\right)$$

The spherical geometry is shown below. The cathode radius is r_c, the anode radius is $\bar{r}_a$:



Generation of electron beams:

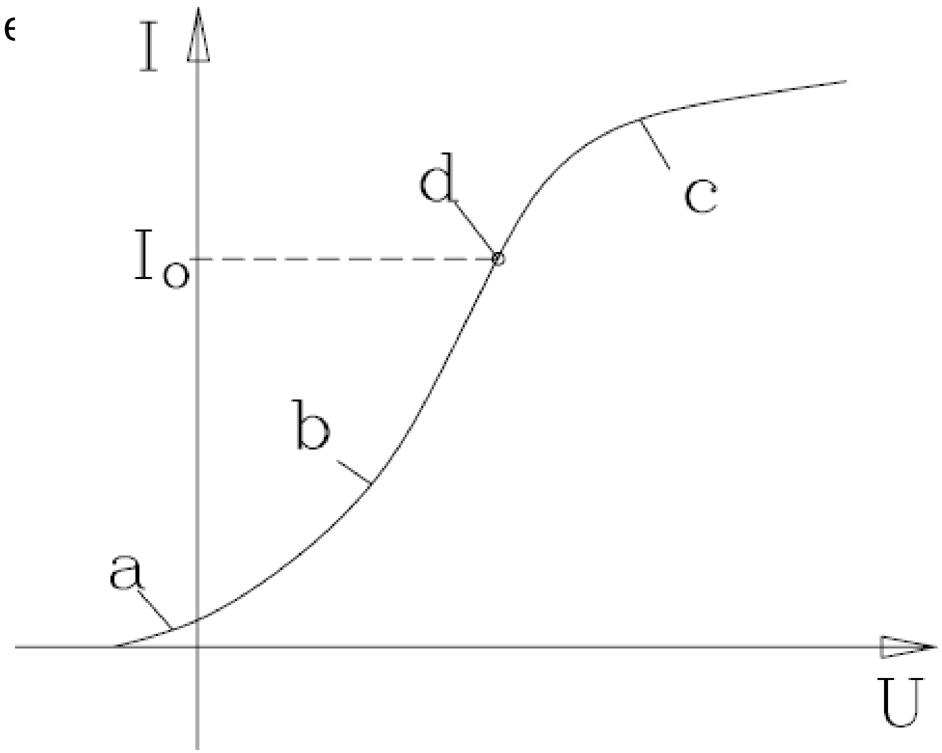
- **thermionic emission**

Richardson-Dushman relation for current density: $j_R = A \cdot b \cdot T^2 \exp\left(-\frac{\phi}{kT}\right) \quad \left[\frac{A}{cm^2}\right]$ (5.8)

Thereby the constant A is $A = \frac{4\pi \cdot m e k^2}{h^3} = 120{,}4 \quad \left[\frac{A}{cm^2 K^2}\right]$

b =material dependent constant
ϕ is the work function, kT the thermal energy of the e
j can only be measured if T >> room temperature

Typical response curve for diode:



The domains are

a) Initial current domain

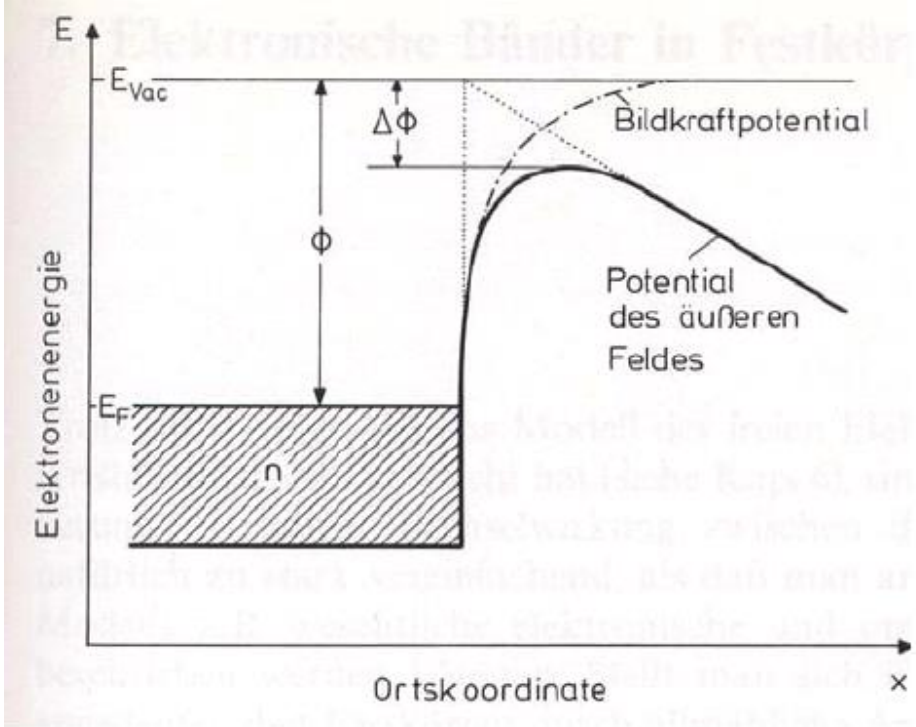
b) Space charge limited regime (Child-Langmuir)

c) Saturation and temperature limited regime

d) is the point were the curve deviates from Child-Langmuir.

- **Field emission**

E-field of the order of 10^7 V/cm
(eg. at cone points, needle cathodes) thereby the potential at the surface of the solid is lowered insofar that the electrons can tunnel.



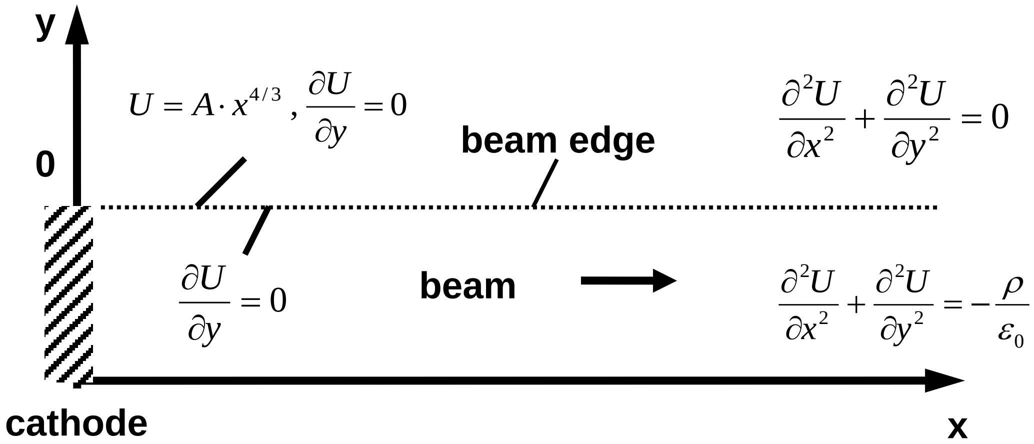
-> Fowler-Nordheim equation:

$$j_{FE} = \frac{K_1 U^2}{\phi} \exp\left(-\frac{K_2 \phi}{U}\right) \quad \left[\frac{A}{cm^2}\right] \qquad (5.9)$$

ϕ is the work function, U the applied voltage and K_1 and K_2 constants

Beam formation in electron guns:

In which way can we shape an electron beam under the influence of space charge? At the beam boundary there is



$$U = A \cdot x^{4/3} \quad \text{and} \quad \frac{\partial U}{\partial y} = -E_y = 0 \qquad (5.10)$$

This is a Cauchy-boundary-condition, at which the field strength is given!

In this case it holds: $U\big|_{y=0} = f(x)\,,\ \left.\frac{\partial U}{\partial y}\right|_{y=0} = 0$

The potential outside the x-axis (y ≠ 0) is the analytical continuation of the function f(x) in the complex plain: $U(x, y) = \mathrm{Re}(f(z)) = \mathrm{Re}(f(x+iy))$ (5.11)

Taylor expansion: $f(x+iy) = f(x) + iy \cdot f'(x) + \frac{(iy)^2}{2!} f''(x) + \cdots$

➔ $U(x, y) = f(x) - \frac{y^2}{2} f''(x) + \cdots \quad , \quad U(x,0) = f(x) \quad , \quad \left.\frac{\partial U}{\partial y}\right|_{y=0} = -y \cdot f''(x)\big|_{y=0} = 0$

Thus, for the beam boundary it is

$$U(x, y) = \mathrm{Re}(A \cdot (x+iy)^{4/3}) = Ar^{4/3}\,\mathrm{Re}(\exp(\tfrac{4}{3} i\varphi)) \qquad (5.12)$$

==> $U(x,y) = A \cdot r^{4/3} \cos(\frac{4}{3}\varphi) = A\left(x^2 + y^2\right)^{2/3} \cos\left(\frac{4}{3}\arctan(\frac{y}{x})\right)$

The cathode potential U_c = 0 one can find with $\cos\left(\frac{4}{3}\varphi\right) = 0$.

Hence, the angle is:

$$\varphi = \frac{3\pi}{8}, \frac{9\pi}{8}$$ or

$$\varphi = 67.5°, 202.5°$$

The cathode boundary needs an angle of 67.5° towards the cathode normal to compensate for the divergence of the space charge dominated beam.

⇒ Boundary cathode with Pierce type boundary

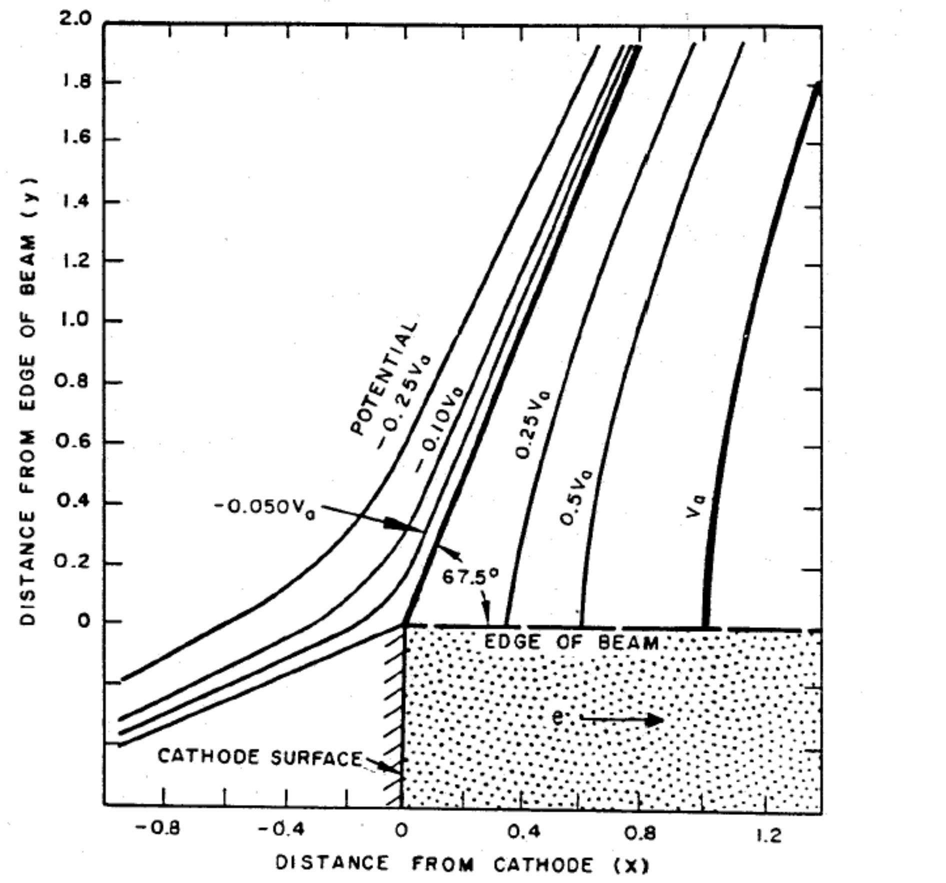
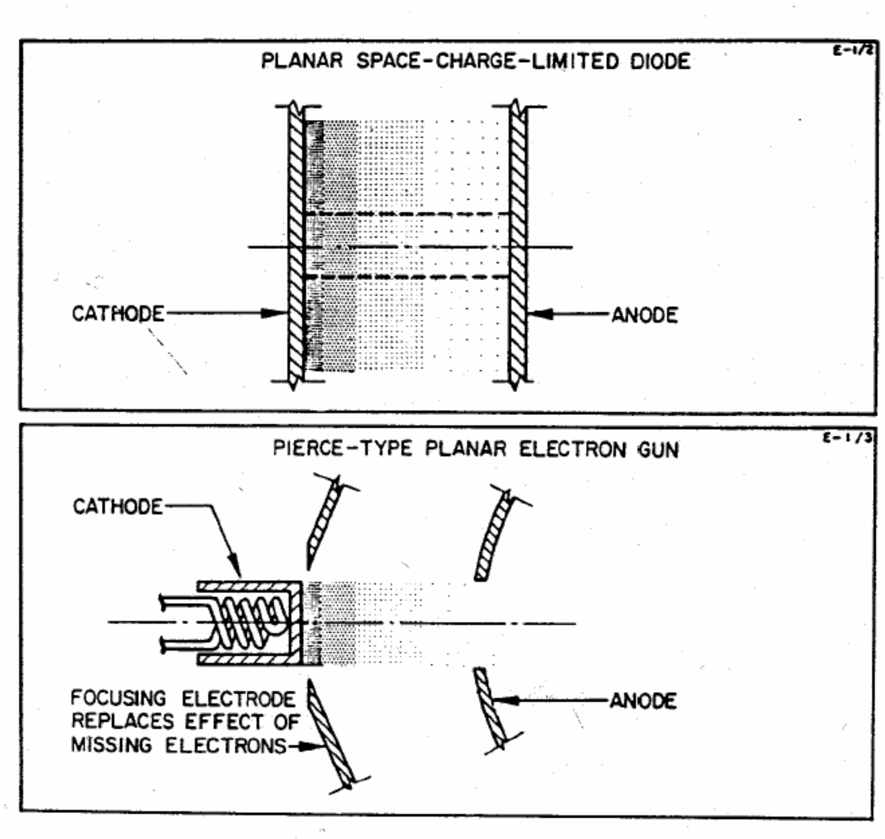
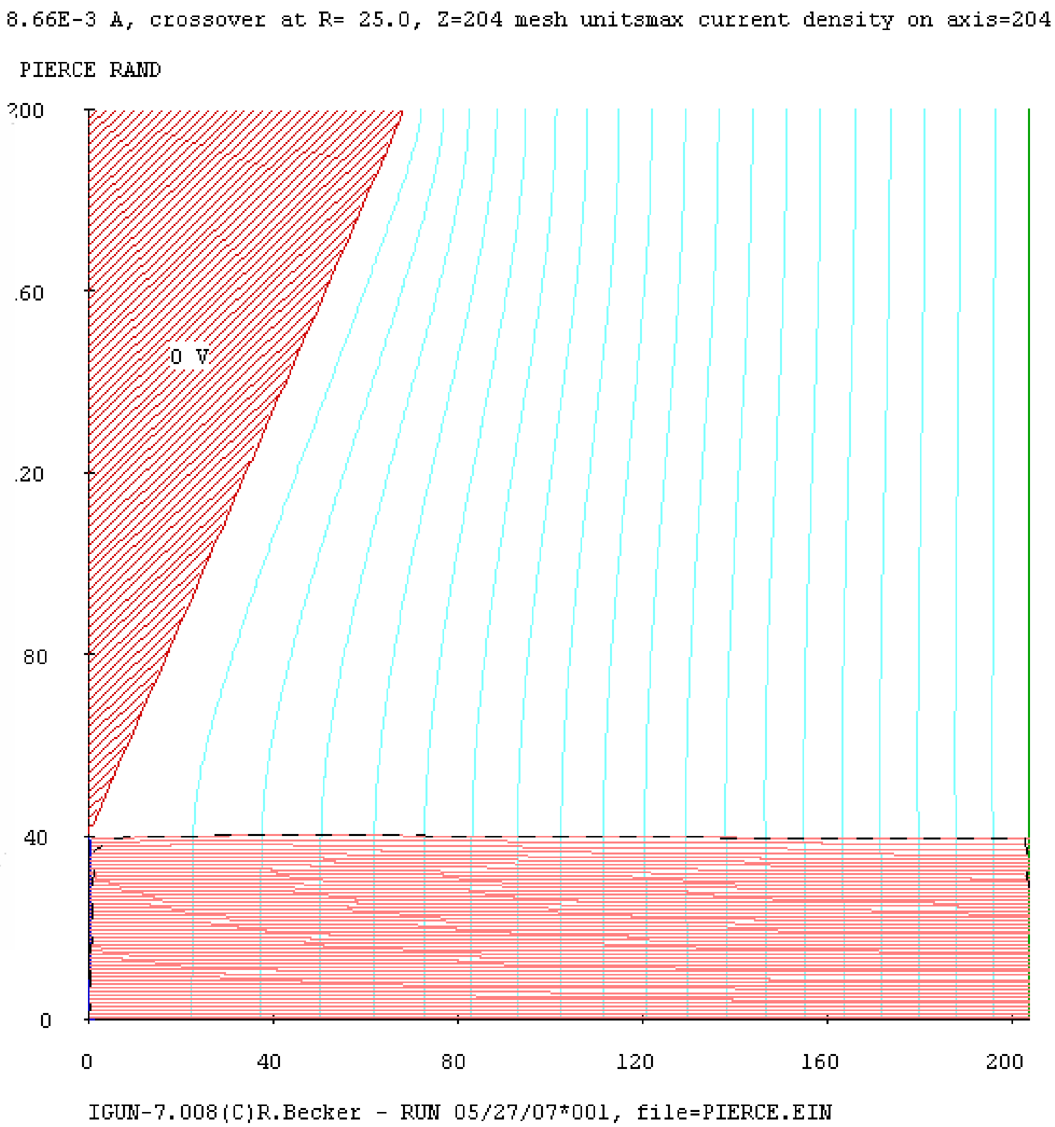


FIG. 4. Plot of the equipotential lines external to a planar space-charge-limited electron beam, as determined from Eq. (7). The heavy lines show the shape of the focus electrode and anode electrode.

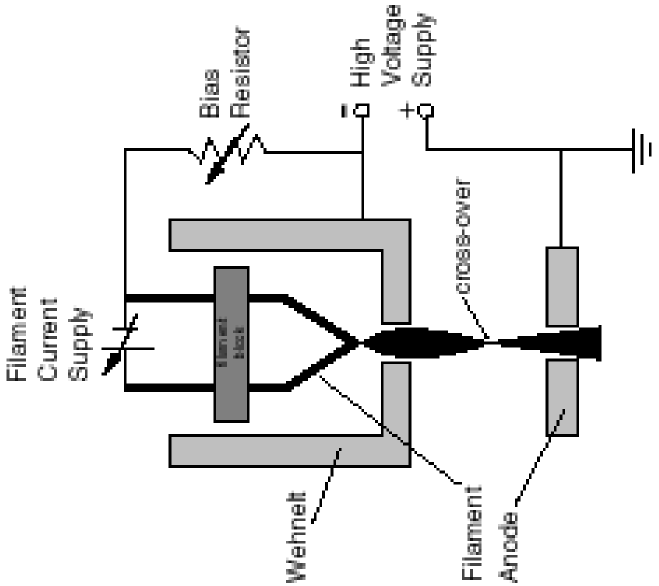
Beam formation of a Pierce-type electron gun calculated for an anode voltage of 1000 V.
1 mm is equal to 4 meshes!

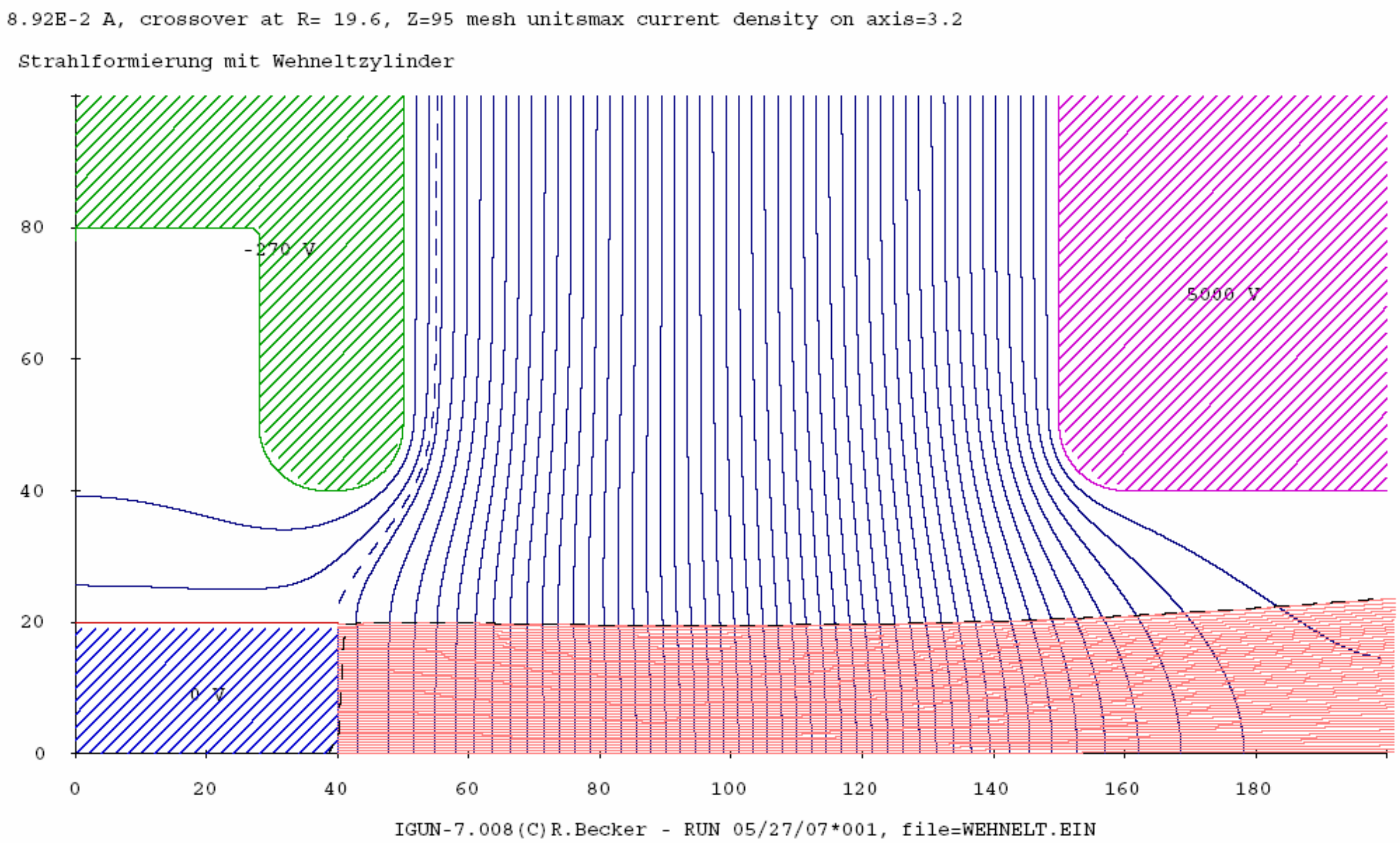




Perveance of such a gun:
$P=2.9*10^{-7}\ A/V^{3/2}$, current I = 9 mA

One can also form the equipotential lines with a Wehnelt cylinder.



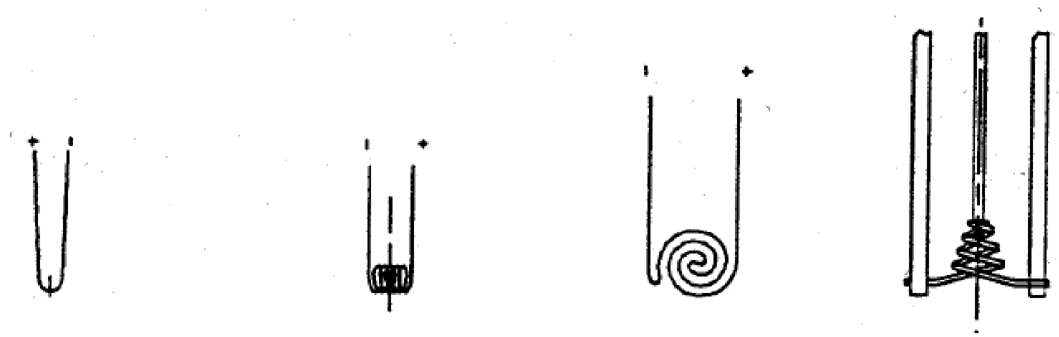


Therewith the potential at the Wehnelt-electrode is adjusted in that way, that the equipotential line, which represent the cathode potential, touches the cathode edge under the Pierce-angle. Wehnelt cylinders are used in electron microscopes and TVs.

Cathodes for E-Guns:

Electrons are emitted from cathodes either via thermionic emission or field emission. The maximum emission current density of the cathode is determined by the Richardson-Dushman equation (5.8). Some examples for cathode material are shown in the table:

Depending on the application, there are different cathode geometries available. The simplest cathodes are made from W- or Tl-wires and are directly heated.



Dispenser cathodes keep a thin film of the emitting material due to supply from a reservoir in a porous matrix usually made from tungsten for instance.

Reservoir materials are Thorium, BaO, CaO…

Work Function and Factor A × b for Various Materials

Material	ϕ(V)	A × b (A cm^{-2} K^{-2})
Molybdenum	4.15	55
Nickel	4.61	30
Tantalum	4.12	60
Tungsten	4.54	60
Barium	2.11	60
Cesium	1.81	160
Iridium	5.40	170
Platinum	5.32	32
Rhenium	4.85	100
Thorium	3.38	70
Ba on W	1.56	1.5
Cs on W	1.36	3.2
Th on W	2.63	3.0
Thoria	2.54	3.0
BaO + SrO	0.95	~10^{-2}
Cs-oxide	0.75	~10^{-2}
TaC	3.14	0.3
LaB_6	2.70	29

Die usual cathode denominations are:

W-matrix	B, S-type
Coated W-matrix	M or CD-type

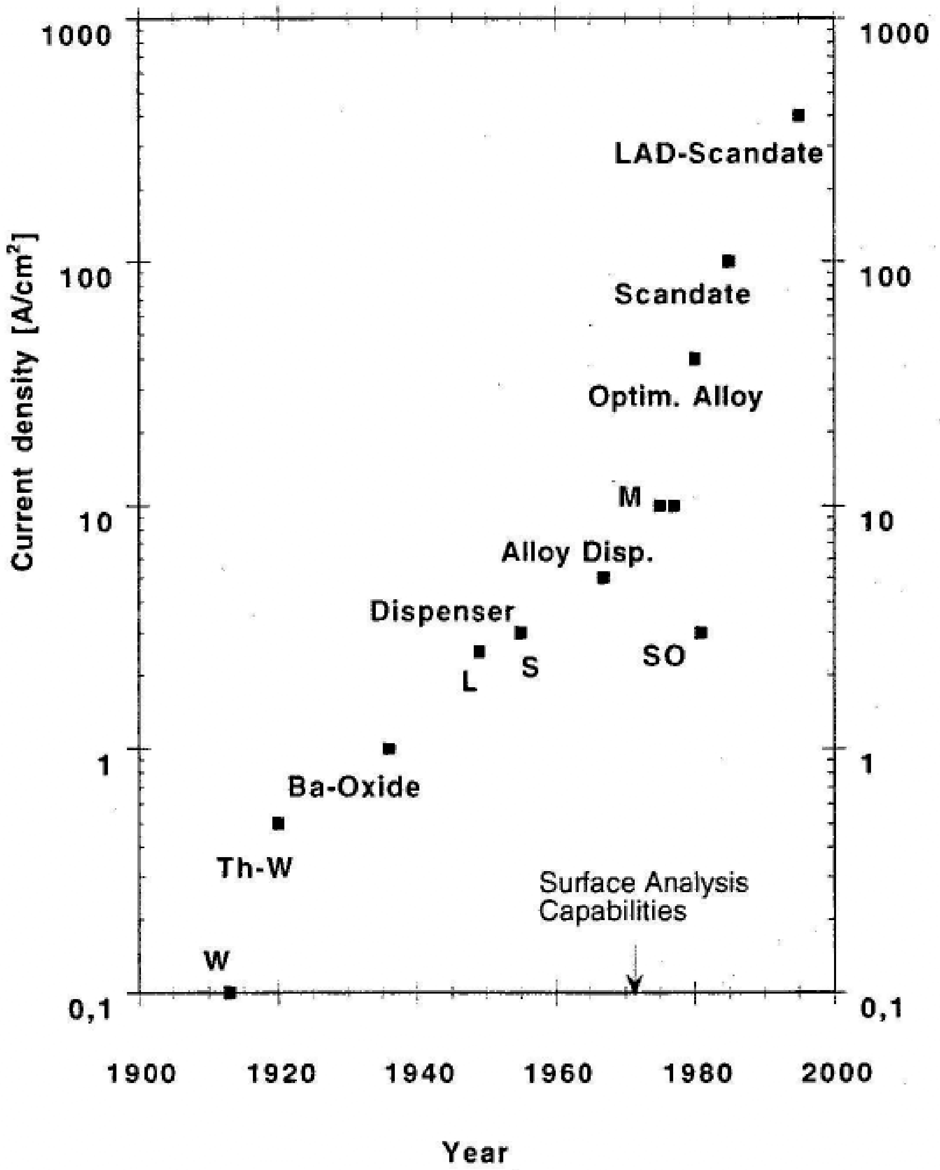
Single crystal-cathodes
LaB_6, IrCe

Dispenser cathodes $j_{max} < 10$ A/cm^2

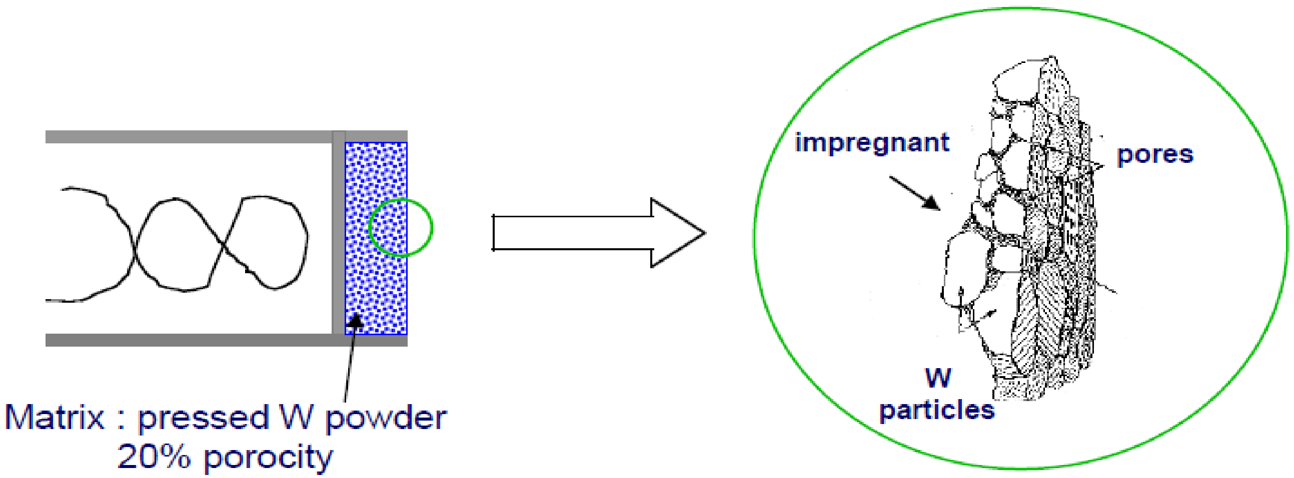
Single crystal-cathode →
j_{max} up to 100 A/cm^2

In front of a dispenser cathode surface a dipole layer from ions of the dispenser material is created, which reduces the work function. Dispenser cathodes are often impregnated with Osmium (M-type) or Scandium. This reduces the work function to 1.4-1.8 eV.

	Φ (eV)	T (J =1 A/cm^2)
Oxide	1.5	942 K
Standard	2.1	1277 K
Os-coated	1.95	1194 K



G.Gartner et al, Appl. Surface Sci 111 (1997) 11.



Impregnant:

4BaO, CaO, Al_2O_3 : [411], S-type

5BaO, 3CaO,2Al_2O_3 : [532], B-type

W provides the electrical conductivity. BaO lowers Φ .

Saturation current density in Dependence on the temperature: