3) Plasma physics and magnetic confinement

To understand ion sources operation, knowledge in plasma physics is required.

The constituents of plasma, depending on the ionization degree, are

Ions, electrons and atoms (neutrals)

Different to a gas, the particles in plasma interact strongly due to Coulomb forces. The behaviour of plasma can be influenced by electric and magnetic field.

The plasma is often referred to the „fourth state of matter! The transition from gas to the plasma state is not a real phase transition in the thermodynamic view, but energy (latent heat) is required to ionize atoms in the gas!

Examples for plasma:

Natural: Sun, ionosphere, polar lights, flames, flashes, e^- in metals

Artifical: Neon tube, plasma screens, plasma etching systems,
fusions plasmas, metal-vapor discharge lamps, ion source plasmas

Due to free moving charged particles in the plasma, it has a high conductivity!

Plasma density und plasma temperature

The plasma density of ions or electrons is determined as particle densities

$$n_i \quad \text{and} \quad n_e \quad \text{in} \quad \left[\frac{1}{cm^3}\right] \quad \text{or} \quad \left[\frac{1}{m^3}\right]$$

A multi component plasma is neutral and the neutrality condition is:

$$\sum q_i \cdot n_i = n_e \tag{3.1}$$

The degree of ionization or fractional ionization is defined as

$$p_{ion} = \frac{n_i}{n_i + n_{neutral}} \tag{3.2}$$

$p_{ion} = 1$ is the case of a completely ionized plasma.

A plasma is called highly ionized if the fractional ionization is larger than 10%.

Typical plasmas in the lab: $n_e \sim 10^{14} – 10^{22}$ 1/m^3

For comparison: Gas density at room temperature and 10^{-2} Pa: $n = 2.5 \cdot 10^{18}$ 1/m^3

The temperature of a plasma is typically given in eV:

1 eV = 11600 K $\qquad e{*}U = kT \rightarrow U = 1\ V \rightarrow T = e/k$

Ion temperature T_i and electron temperature T_e don't have to be identical. In presence of a magnetic field, an anisotropy is introduced, leading to different T parallel and perpendicular to the field:

$$\Rightarrow\ T_{i\parallel}, T_{i\perp}, T_{e\parallel}, T_{e\perp}$$

The reason for that is the different mobility of the particles parallel and perpendicular to the field.
The concept of temperature is also applied to plasma, which are ***not*** in thermal equilibrium!

Typical electron temperatures:

- surface ionized plasma $\qquad T_e \sim 0.2$ eV
- arc discharge plasma $\qquad T_e \sim 1$ eV
- microwave generated plasma $\qquad T_e \sim$ some keV, $T_i < 1$ eV

Plasma frequency
The electrons as well as the ions can oscillate inside the plasma. Depending on the component different modes can be present, electron-plasma oscillation (Langmuir wave) or "ion sound".

Continuity equation: $\frac{\partial \rho}{\partial t} + div\,\vec{j} = 0 \qquad \vec{j} = \rho \cdot \vec{v}, \rho = n_e \cdot m_e$ (3.3)

The motion of plasma is implied by Euler's equation (hydrodynamic equation of motion for incompressible and frictionless flow)

$$\rho \cdot \left(\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} \right) = \rho \cdot \frac{d\vec{v}}{dt} = \vec{F} \quad with \quad \vec{F} = -n_e e \vec{E} \tag{3.4}$$

Divide the charge carrier density n_e into a constant part n_{e0} and a flow n_{e1} with

$n_{e0} >> n_{e1}$, in which n_{e1} is not neutralized!

Due to the quasi-neutrality $\rho_{el} = -e \cdot n_{e0} + \sum_i q_i n_{i0} - e \cdot n_{e1} = -e \cdot n_{e1}$ (3.5)

From the 1st Maxwell equation follows $div\,\vec{E} = \frac{\rho_{el}}{\varepsilon_0} = -\frac{e \cdot n_{e1}}{\varepsilon_0}$

Assumption: $\vec{v}, n_{e1}\ und\ \vec{E}$ are small numbers → linearization of equation is possible:

$$\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m_e}\vec{E}$$ from the Euler equation and $$\frac{\partial n_{e1}}{\partial t} + n_{e0} \cdot div\,\vec{v} = 0$$ from the continuity equation.

With both equations one get:

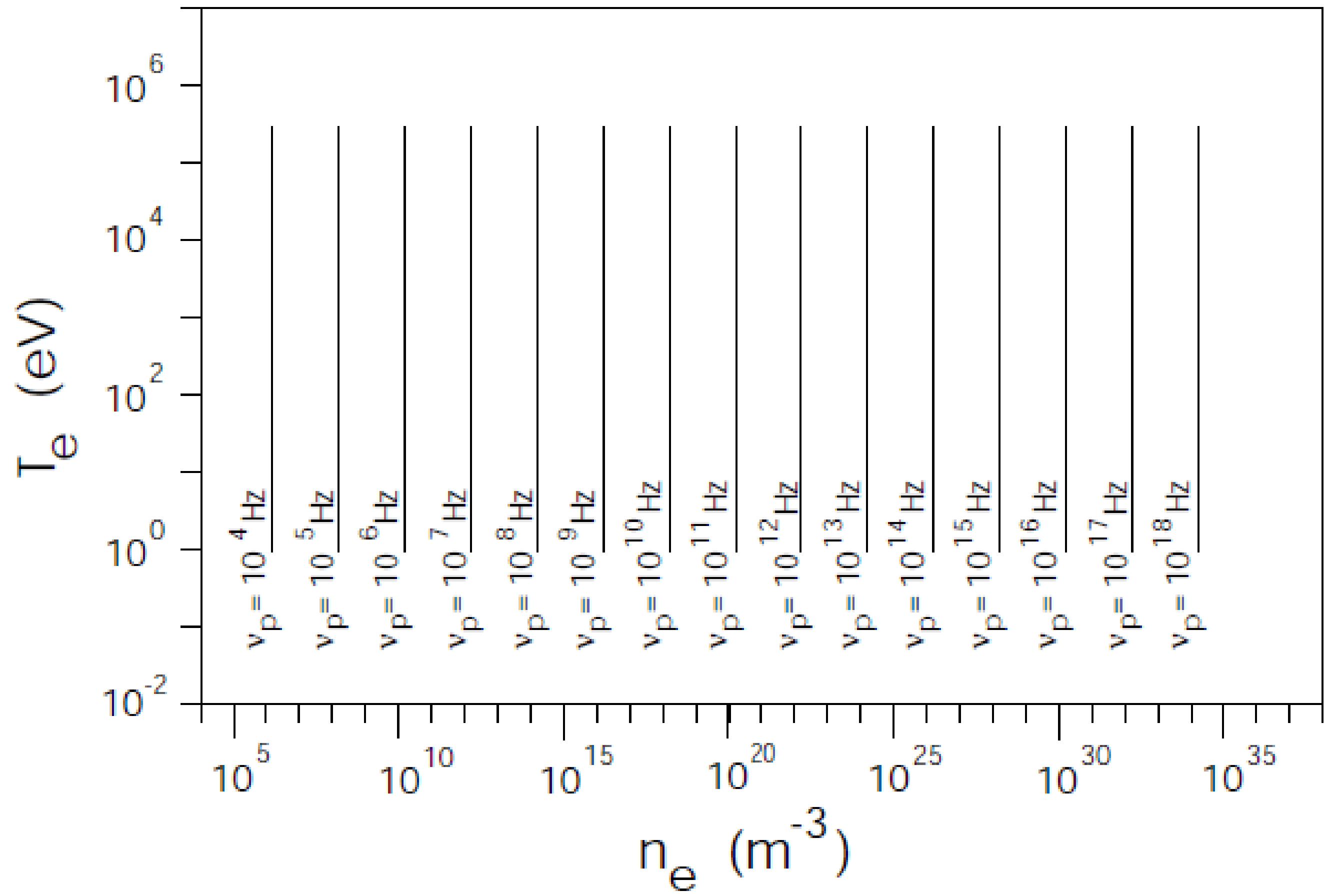
$$\frac{\partial^2 n_{e1}}{\partial t^2} + n_{e0}\frac{e^2 n_{e1}}{m_e \varepsilon_0} = 0 \qquad (3.6)$$

oscillation equation with $n_{e1}(t) = A \cdot \cos \omega_p t$

➔ $$\omega_p^2 = \frac{e^2 \cdot n_{e0}}{m_e \cdot \varepsilon_0}$$ **Plasma frequency** (3.7)

$1/\omega_p$ is the characteristic time, in which the plasma reacts on a disturbance!

$$f_p = \frac{1}{2\pi}\sqrt{\frac{e^2}{\varepsilon_0 m_e}} \cdot \sqrt{n_e} = 8{,}98 \cdot \sqrt{n_e[\tfrac{1}{m^3}]}\,[Hz] \qquad (3.8)$$



Only electromagnetic waves with $f > f_p$ are able to travel through the plasma. ω_p or f_p is called the cut-off-frequency of the plasma.

The calculation of the electric field of a disturbance shows that the disturbance can only be small.

Assuming a disturbance of 1% in the quasi-neutrality in a local environment of 1mm size:

$$div\,\vec{E} = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad \frac{\delta E}{\delta x} \approx \frac{0.01 \cdot n_e \cdot e}{\varepsilon_0} \quad \Rightarrow \quad \delta E \approx 10^{10}\,\frac{V}{m}$$

Such a field can't be preserved. Thus, quasi-neutrality is given, with only small disturbances. The electron density at which an electro-magnetic wave is reflected is called the critical density

$$n_{critical} = \frac{4\pi^2 \varepsilon_0 m_e}{e^2} \cdot f_p^2 = 0.0124 \cdot \left(f_p [Hz] \right)^2 \left[\frac{1}{m^3} \right] \tag{3.9}$$

Example for cut-off frequencies:

- Free electrons in metal, density ~10^{28} 1/m^3 →f ~ 10^{15} Hz, visible light is reflected, UV-light can pass.
- Ionosphere, density ~10^9 – 10^{10} 1/m^3 → f ~10^5 –10^6 Hz, medium wave is reflected, metric wave can pass.

The Debye-length

The quasi-neutrality causes a shielding of plasma particles among each other. The region of the shielding can be calculated via perturbation calculation:

$$n_e(\vec{r}) = \bar{n}_e + n_{e1}(\vec{r}) \quad ; \quad n_i(\vec{r}) = \bar{n}_i + n_{i1}(\vec{r})$$

Poisson equation $$\Delta\phi = -\frac{\rho}{\varepsilon_0} = -\frac{1}{\varepsilon_0}\left[-e\cdot\overline{n}_e + Ze\cdot\overline{n}_i - e\cdot n_{e1}(\vec{r}) + Ze\cdot n_{i1}(\vec{r})\right]$$

With the assumption of thermal equilibrium (T_e=T_i and Boltzmann distribution)

$$n_{e1}(\vec{r}) = \overline{n_e}\cdot\exp\left[\frac{e\phi(\vec{r})}{kT}\right] \quad , \qquad n_{i1}(\vec{r}) = \overline{n_i}\cdot\exp\left[\frac{Ze\phi(\vec{r})}{kT}\right] \tag{3.10}$$

and quasi-neutrality, one can obtain a linearization of (3.3) and insertion in Poisson equation delivers:

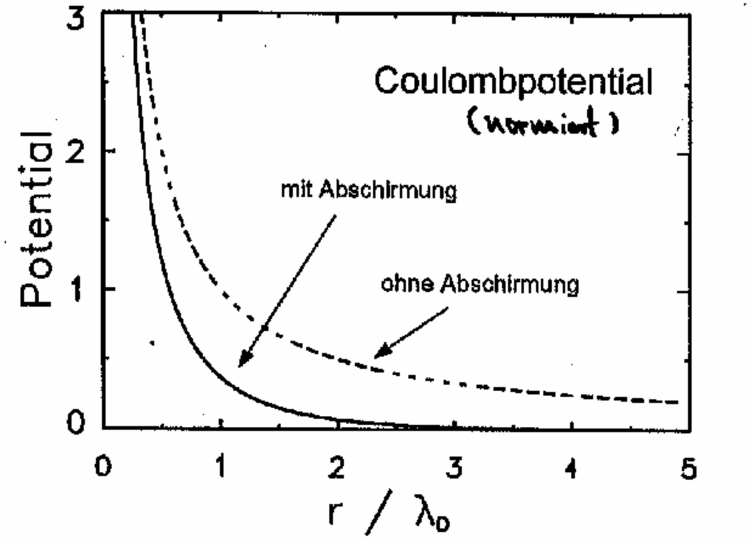
$$\Delta\phi(\vec{r}) - \frac{e^2\cdot\overline{n}_e}{\varepsilon_0}\left[\frac{1}{kT_e} + \frac{Z}{kT_i}\right]\phi(\vec{r}) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\phi(r)\right) - \frac{2e^2\cdot\overline{n}_e}{\varepsilon_0 kT_e}\phi(r) = -\frac{\rho_0}{\varepsilon_0} \tag{3.11}$$

Solution: $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}e^{-\frac{\sqrt{2}r}{\lambda_D}}\cdot\hat{\vec{r}}$ for $\rho_0(\vec{r}) = q\cdot\delta(\vec{r})$ (for a single charge q)

➔ $$\lambda_D^2 = \frac{\varepsilon_0\cdot k\cdot T_e}{\overline{n}_e\cdot e^2} \tag{3.12}$$ **Debye-length**

The Debye-length is the characteristic length for the shielding of constant background charge density ρ_0.

The solution of the differential equation above is the Coulomb potential with shielding.



The function is shown by the following graph.

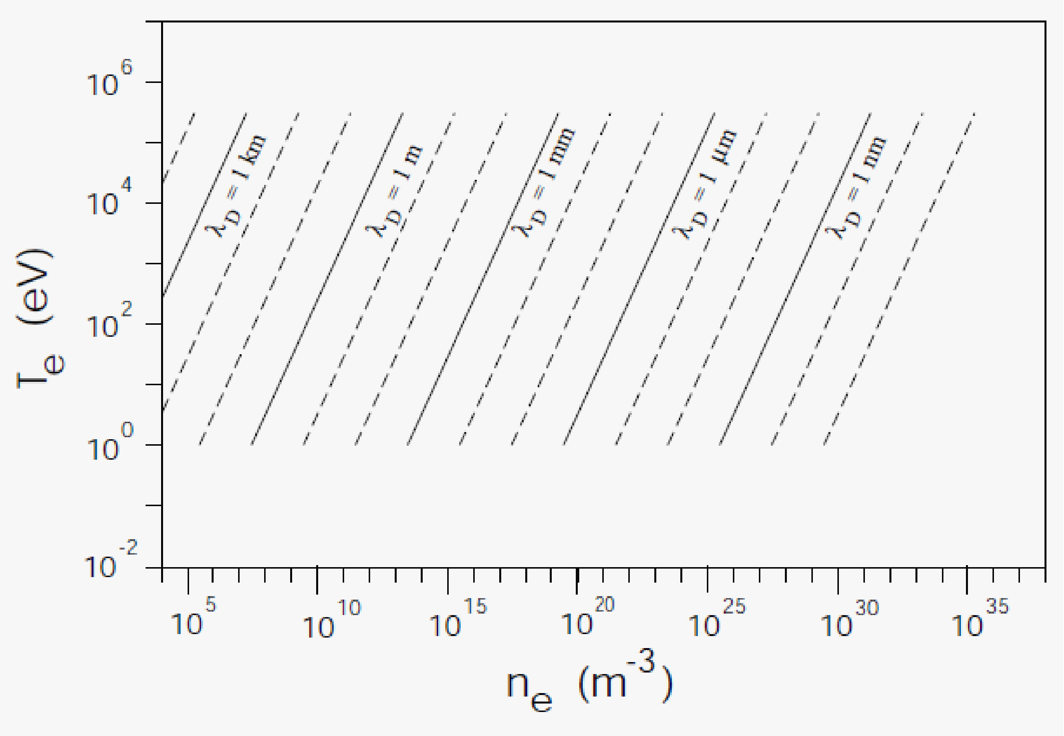
For r > λ_D the potential drops with as e-function

Each charged particle is surrounded by a Debye-cloud

Calculation of Debye-length:

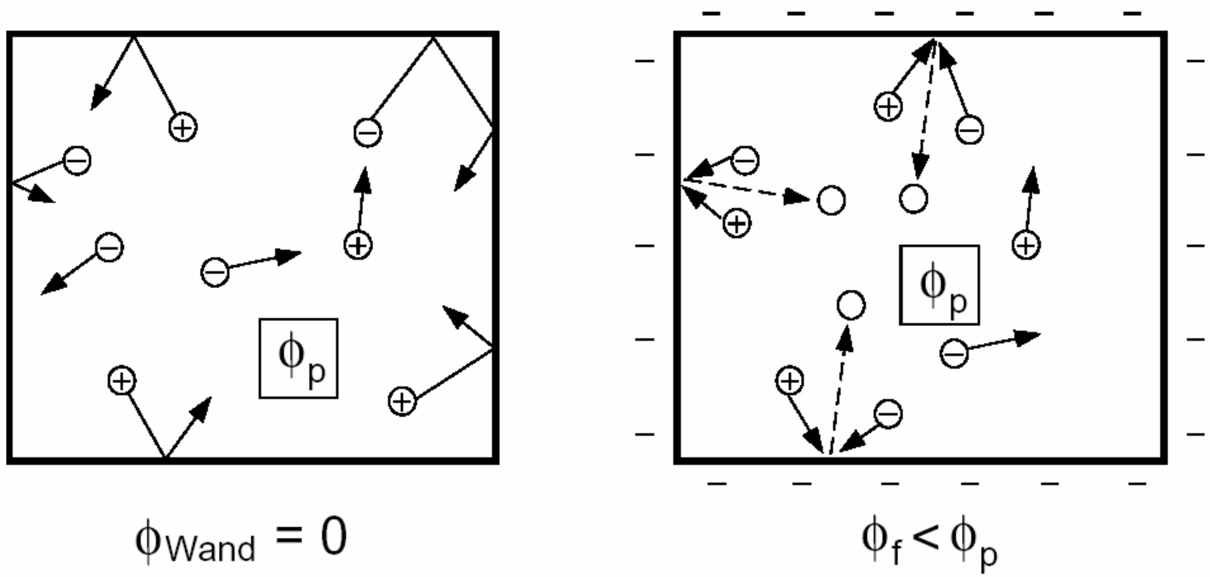
$$\lambda_D = \sqrt{\frac{\varepsilon_0 \cdot k \cdot T_e}{\bar{n}_e \cdot e^2}}$$

$$\lambda_D = 7437 \sqrt{\frac{T_e[eV]}{\bar{n}_e[\frac{1}{m^3}]}}\,[m] \qquad (3.13)$$



Plasma edge and plasma potential

A plasma appears as quasi-neutral on a scale x > λ_D. The potential inside the plasma with respect to the wall is constant and is called the plasma potential ϕ_p. If plasma particles would be elastically reflected by the wall, ϕ_p would be equal to the wall potential. The particles recombine with a probability of 99%.

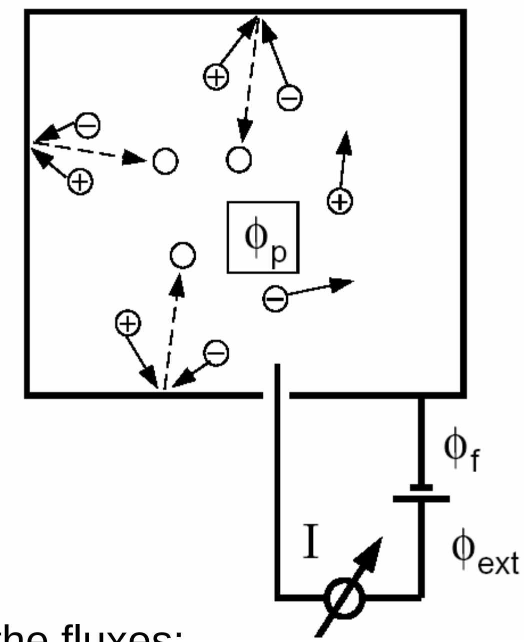


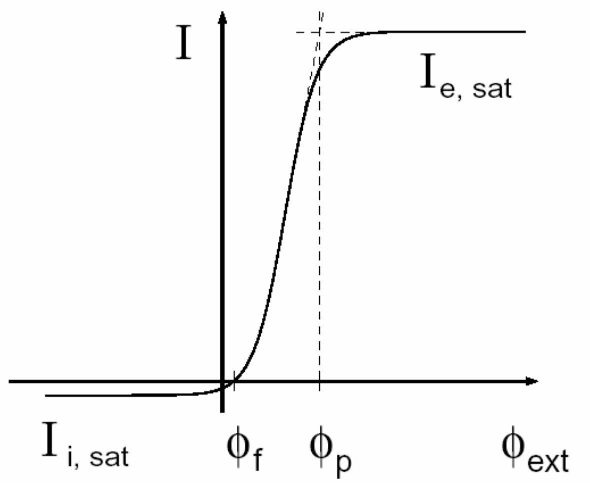
Plasma-wall interface: a) ideally reflected, b) complete wall recombination

Flux is proportional to the mean velocity of the particles. Therefore $\Gamma_e = \sqrt{\frac{m_i}{m_e}}\Gamma_i$ (3.14)

Given that the velocity, and thereby the flux of electrons, is higher compared to the ion flux, a current would break the quasi-neutrality. Thus the wall becomes negatively charged with respect to the plasma or the plasma positive against the wall. Hence the electrons are decelerated and the ions accelerated which balances the net currents. A charge double-layer develops which can be in the order of the Debye-length. The layer is called Debye-layer. The thickness of the plasma edge depends on the potential of an eventually existing electrode with $\phi >> \phi_p$.

An insulated electrode inside the plasma will be charged up to a „floating potential“ ϕ_f, which is about 3-4 times kT_e. For this electrode $\Gamma_e = \Gamma_i$ holds true, i.e. there is not net current.





With a probe reaching in to the plasma, which is connected to a power supply, it is possible to measure the potential.

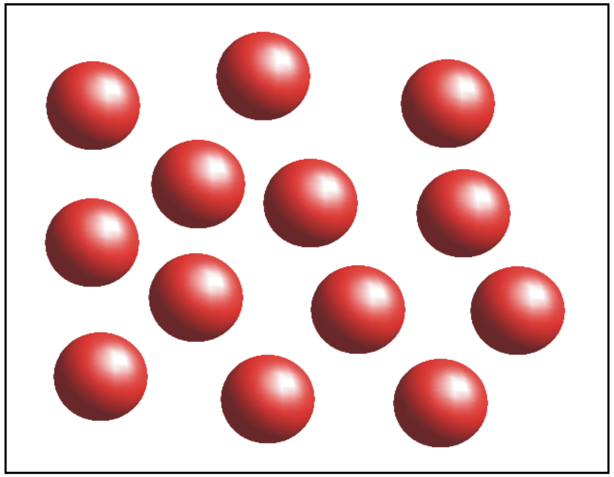
Such a probe is called

Langmuir-probe.

By varying the potential of the probe with respect to the wall, the characteristic current curve can be obtained.
The saturation currents are proportional to the fluxes:
The application of the Langmuir-probe in practice is limited by the power dissipated on the probe.

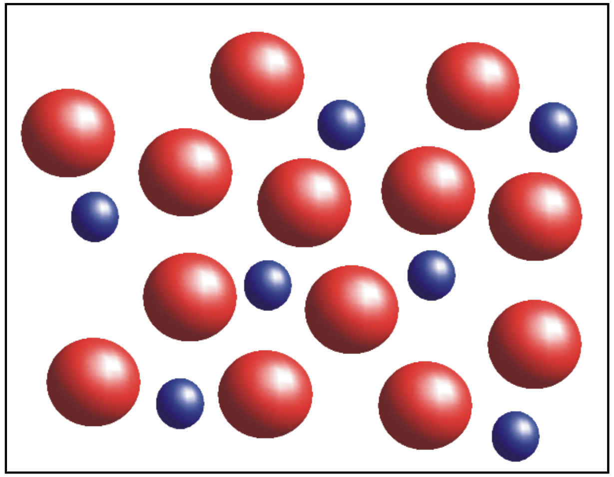
Generation of ion source plasma

Erzeugung eines quasineutralen Plasmas durch Stoßionisation $\rightarrow \sum q_i n_i = n_e$



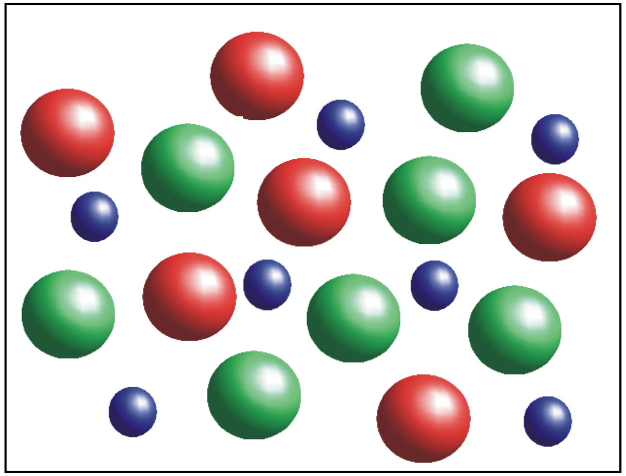
Bereitstellung freier Atome im Plasmagenerator durch:

- Einlassen eines Arbeitsgases
- Schmelzen und Verdampfen
- Sputtern von Feststoffen



Bereitstellung freier Elektronen durch:

- Glühemission
- Photoionisation
- Funkenentladung

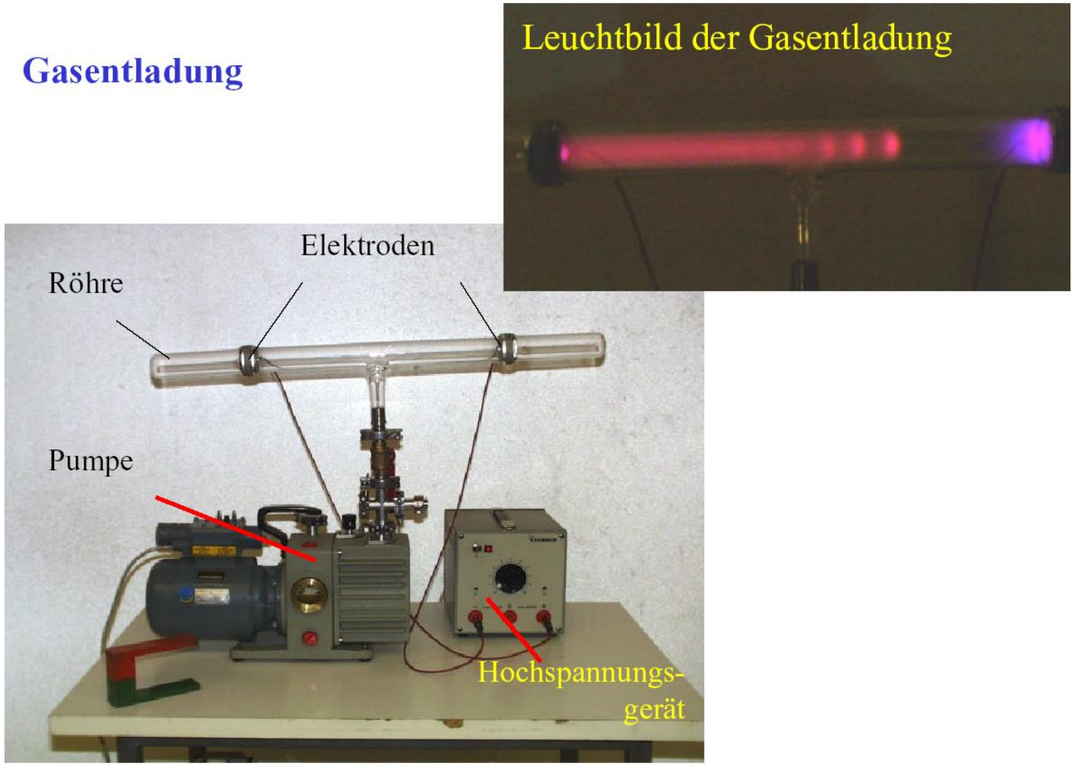


Bereitstellung der Ionisations-energie durch:

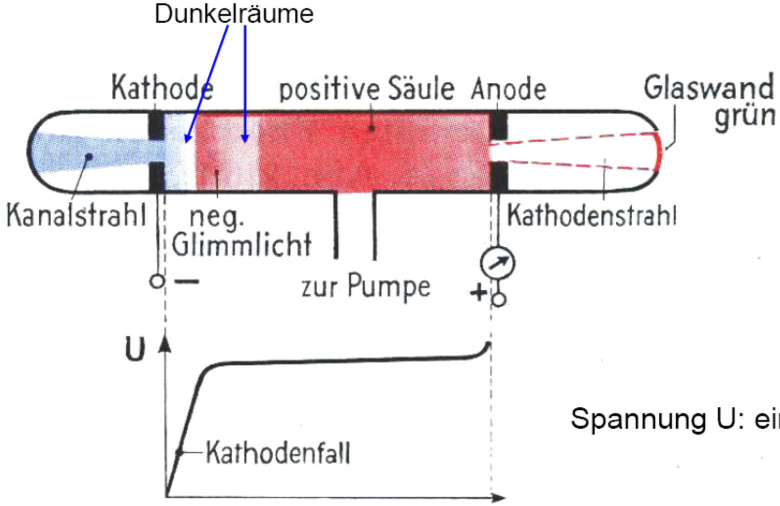
- Beschleunigung der Elektronen
- HF-Heizung
- E x B-Drift

A simple experiment to generate a plasma is the gas discharge. For the experiment a vacuum tube is needed. The following figures show the simple experimental setup. By lowering the gas pressure inside the tube a glow of the cathode can be recognized. The neutral gas is then partially converted to plasma (it became conductive).

Gasentladung



Gasentladungen



Spannung U: einige k

214.1 Glimmentladung mit Spannungsverlauf

Spannungsverlauf wird bedingt durch positive Raumladung (Ionen) im Dunkelraum.

nach: Dorn, Physik Oberstufe, Schroedel Verlag, Hannover 1971

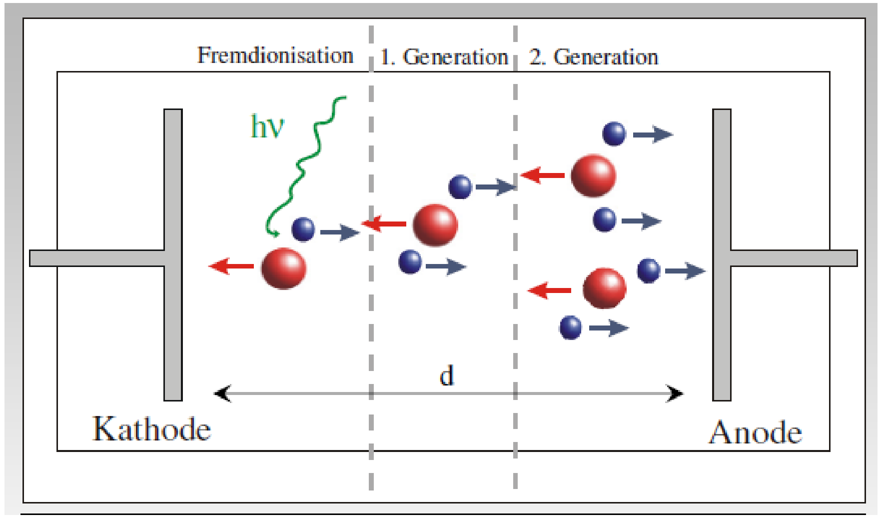
In the neutral gas, atoms can by ionized (e.g. by X-rays from the environment) and thereby electrons are released. The current between anode and cathode depends on the amount of charges generated on the electrons path.

$$\alpha = \frac{\text{Probability for ionization by } e^-}{\text{path length}} \qquad \text{first Townsend coefficient} \qquad (3.15)$$

Below a certain gas pressure p a stationary discharge emerges, i.e. electrons are continuously generated. Impacts of the ions onto the cathode material releases secondary electrons.
The number of secondary electrons γ per ion impact is given by the third Townsend coefficient with

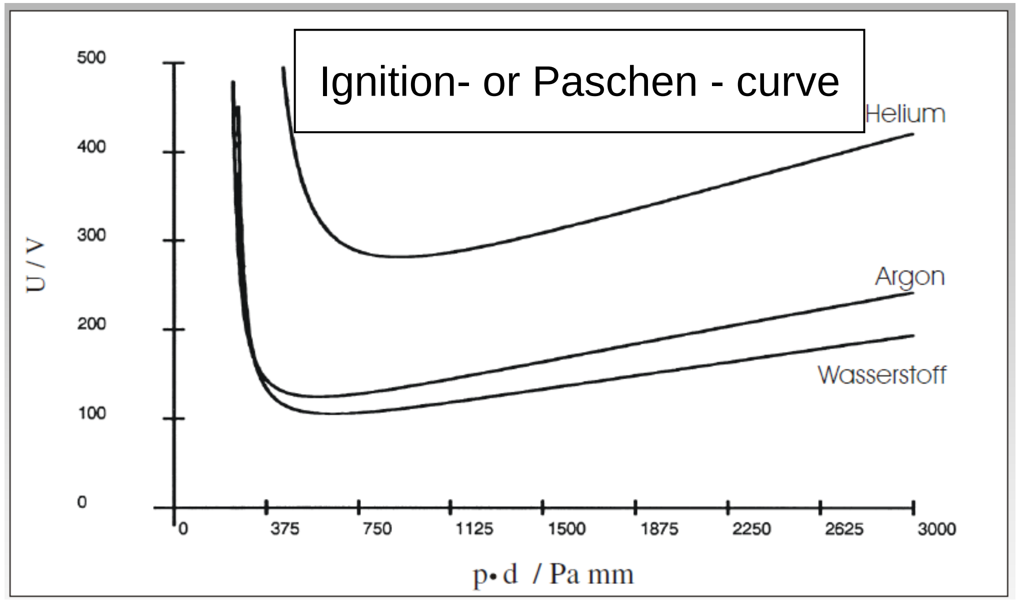
$$dN_e = N_i \cdot \gamma \qquad \gamma \approx 0.01 - 0.1$$

ignition condition $$\gamma \cdot e^{\alpha d} \geq 1 \qquad (3.16)$$



From the ignition condition follows the

$$U_{Ignition} = \frac{c_2 p \cdot d}{\ln(c_1 p \cdot d) - \ln\left(\ln\left(\frac{1}{\gamma}\right)\right)} \qquad (3.17)$$



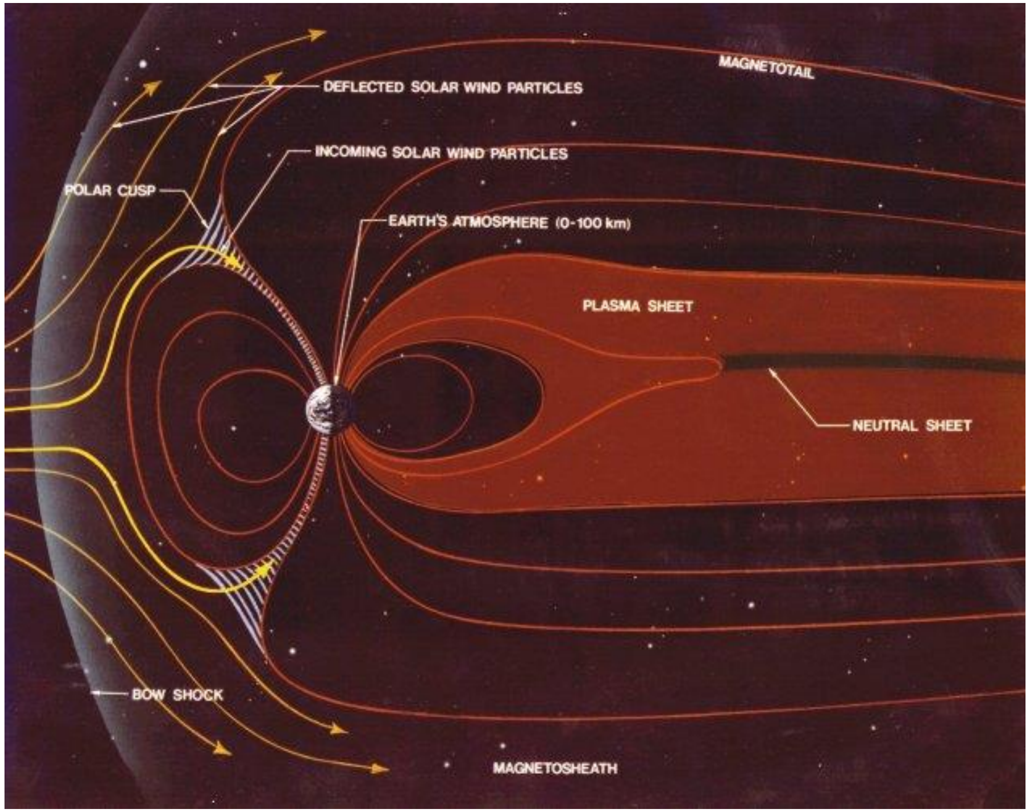
For high p*d the short mean free path has to be compensated by a higher U.

For small p*d, U → ∞, because the mean free path becomes >> d. For the minimum of the curve, the mean free path equals ~d. The difference between the curves arises from specific secondary-electron yield γ for each element.

Magnetic confinement of plasma

Due to the shielding in the plasma, there is almost no control with electric fields. A control is possible with magnetic fields, as particles are bound to the field lines. This is called plasma confinement. It is of fundamental interest for the magnetic confinement fusion (Tokamak, Stellarator).

Example in nature: Guidance of High energetic particles of the cosmic radiation by the earth magnetic field to the poles → generation of polar lights.



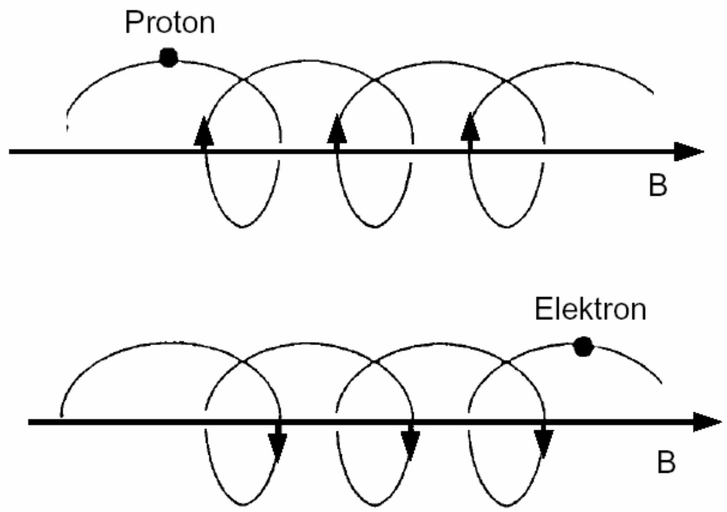
We solely take single particle motion in the magnetic field into account. If we include collective effects, we need magneto hydrodynamik (MHD) theory.

Lorentz force: $m \cdot \dot{\vec{v}} = \vec{F}_L = q \cdot \vec{v} \times \vec{B}$

As $\vec{F}_L \perp \vec{B}$ yields $m \cdot \dot{\vec{v}}_\parallel = 0$, the magnetic field has no influence on the motion parallel to the magnetic field. The direction of the velocity changes perpendicular to the magnetic field, hence the kinetic energy of the particles is constant (orbit motion).

$$m \cdot \frac{v_\perp^2}{r} = m\omega_c^2 r = q \cdot v_\perp \cdot B = q\omega_c r \cdot B$$

→ $\omega_c = \dfrac{q}{m} B$ **cyclotron frequency** (3.18)



$$\omega_{ce}[Hz] = 1.76 \cdot 10^{11} \cdot B[T] \quad , \quad \omega_{ci} = \frac{m_e}{m_i}\omega_{ce} \, ,$$

Radius on orbit: $r = \dfrac{m \cdot v_\perp}{q \cdot B}$ (3.19)

Injection of an electromagnetic wave of $\omega = \omega_{ce}$ → plasma heating

Using $\frac{1}{2} m \cdot v_\perp^2 = kT$ (2 degrees of freedom perpendicular to B) the Lamor radius r_L is

$$r_L = \frac{m \cdot v_\perp}{q \cdot B} = \frac{\sqrt{2mkT}}{qB} = 3.4 \cdot 10^{-6} \cdot \frac{\sqrt{T_e[eV]}}{B[T]} [m] \qquad (3.20)$$

Example: T_e = 1 eV, B = 1 T $\rightarrow$ r_L = 3.4 μm

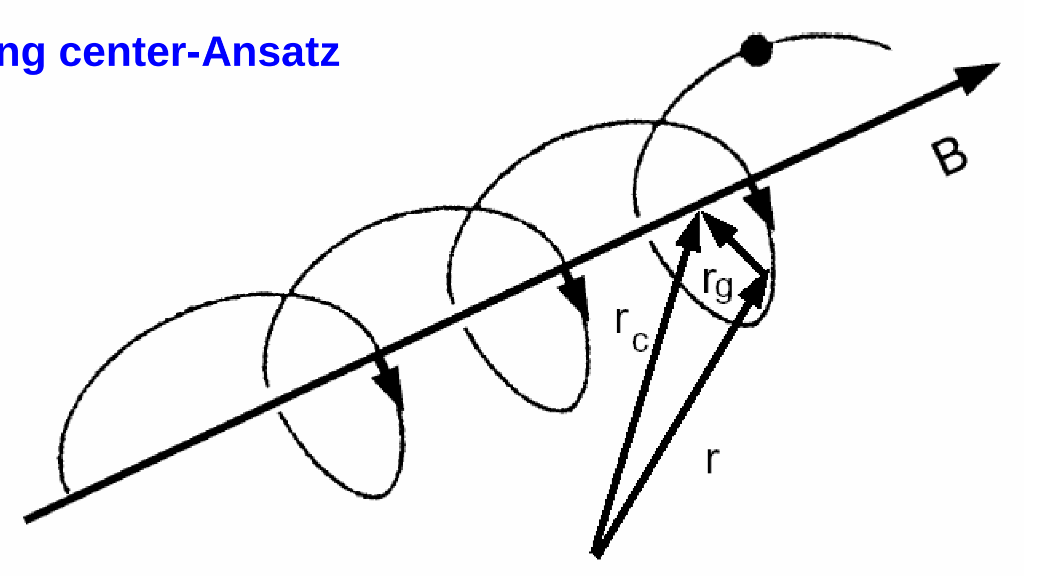
Let's now investigate $m \cdot \dot{\vec{v}} = \vec{F} + q \cdot \vec{v} \times \vec{B}$

Assuming an additional force F acting on the particles ⇨ no closed orbit

For constant B und F in time and space → **guiding center-Ansatz**

The center motion is calculated via

$$\vec{r}_c = \vec{r} + \vec{r}_g \ , \quad \vec{r}_g = \frac{m \cdot v}{q \cdot B} \cdot \frac{\vec{v} \times \vec{B}}{v \cdot B} = \frac{m}{q \cdot B^2} \vec{v} \times \vec{B}$$



The velocity of the guiding center is given by

$$\vec{v}_c = \dot{\vec{r}}_c = \dot{\vec{r}} + \dot{\vec{r}}_g = \vec{v} + \frac{m}{q \cdot B^2} \cdot \dot{\vec{v}} \times \vec{B} = \vec{v} + \frac{1}{q \cdot B^2} (\vec{F} + q \cdot \vec{v} \times \vec{B}) \times \vec{B}$$

with $(\vec{v} \times \vec{B}) \times \vec{B} = (\vec{v} \cdot \vec{B})\vec{B} - B^2\vec{v} = -B^2\vec{v}_\perp$ the following result of the motion is derived

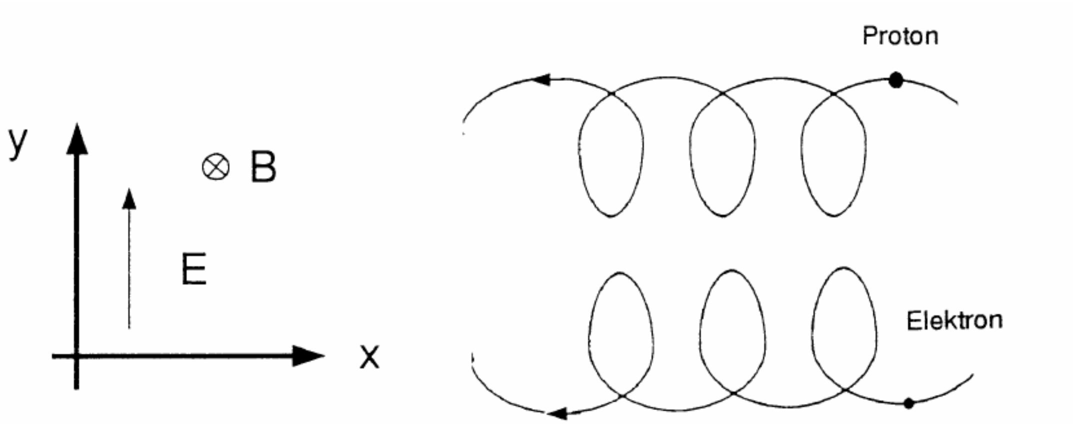
$$\vec{v}_c = \vec{v}_\parallel + \frac{\vec{F} \times \vec{B}}{q \cdot B^2} \quad \text{and} \quad m\dot{\vec{v}}_\parallel = \vec{F}_\parallel \qquad (3.21)$$

The second term is called drift velocity. This term does not accelerate and is directed prependicular to B and $F_\perp$.

Examples: **1.) electric field** $\vec{F} = q \cdot \vec{E}$ → E x B –drift

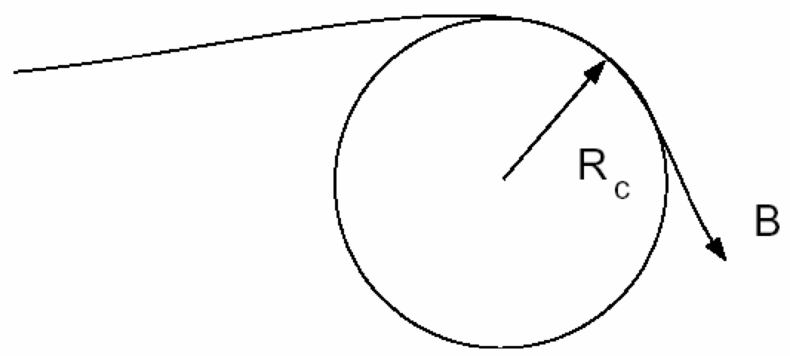
→ $\vec{v}_D = \dfrac{\vec{E} \times \vec{B}}{B^2}$

The drift velocity is not dependent on the charge of the particle.



ExB-drift in direction of the x-axis. The gyration radii are not drawn to scale!

2.) $\vec{\nabla}$ **B-Drift**, if $\vec{B} = \vec{f}(x,t)$ → curvature drift



Due to $\vec{\nabla} \cdot \vec{B} = 0$ a gradient in B leads to a curvature of the field lines. This results in a

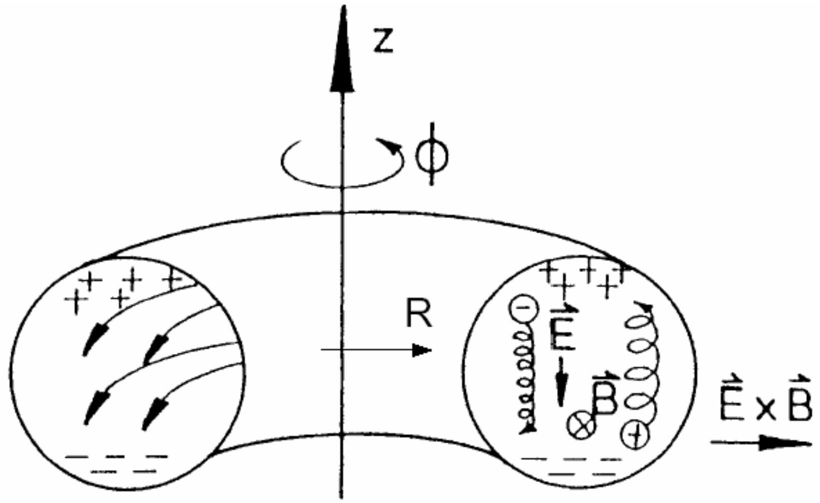
centrifugal force $\vec{F} = m \cdot \frac{\left|\vec{v}_\parallel\right|^2}{R_c} \cdot \hat{\vec{r}}_c$.

$$\vec{v}_D = \frac{m \cdot \left|\vec{v}_\parallel\right|^2}{R_c} \cdot \frac{\hat{\vec{r}}_c \times \vec{B}}{q \cdot B^2}$$

$$\frac{\hat{\vec{r}}_c}{R_c} = \frac{d\hat{\vec{T}}}{ds} = -\frac{\vec{\nabla} B}{B}$$

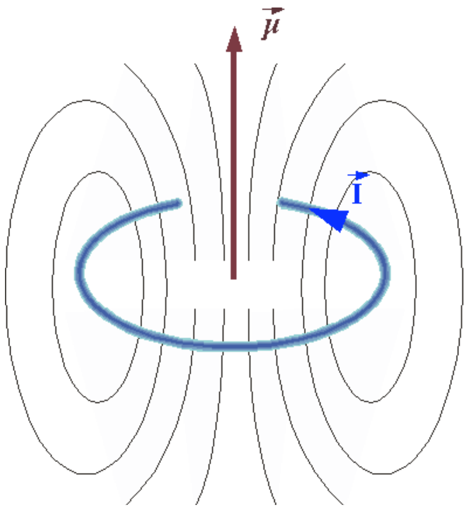
The centrifugal force is in direction of $-\vec{\nabla} B$ and hence

$$\vec{v}_D = -\frac{m \cdot \left|\vec{v}_\parallel\right|^2}{q \cdot B^3} \cdot \vec{\nabla} B \times \vec{B}$$



Particle drift in a toroidal magnetic field. The drift results in a charge separation and the resulting ExB-drift moves the plasma outward

Adiabatic invariance of the magnetic moment



A charged particle on a circular orbit creates a current in the magnetic field.

$$I = q\frac{v}{2\pi \cdot r} = q\frac{\omega}{2\pi}$$

magnetic dipole moment:

$$\vec{\mu}_m = I \cdot \int_F dF \cdot \vec{n}_F = I \cdot F \cdot \vec{n}_F = \tfrac{1}{2} q\omega_c \cdot r_L^2 \cdot \vec{n}_F = \frac{q^2}{2m} B \cdot r_L^2 \cdot \vec{n}_F$$

Assuming the Lamor radius $r_L = \frac{v_\perp m}{B \cdot q}$ ➔ $\mu_m = \frac{q^2}{2m} B \cdot \left(\frac{v_\perp m}{B \cdot q}\right)^2 = \frac{m \cdot v_\perp^2}{2B} = \frac{W_\perp}{B}$ (3.22)

How does the magnetic moment change with a slow drift of the guiding centre?
Change of the magnetic flux ϕ through the orbit area!

Gyration period $\tau_C = \frac{2\pi}{\omega_c}$ and $\Delta W_\perp = q \cdot \frac{d\phi}{dt} = \pi \cdot q \cdot r_L^2 \cdot \frac{dB}{dt}$

→ $$\frac{dW_\perp}{dt} \approx \frac{\Delta W_\perp}{\tau_c} = \frac{\pi \cdot q \cdot r_L}{\tau_c} \cdot \frac{dB}{dt} = \mu_m \frac{dB}{dt}$$ (3.23)

if τ_c and r_L are introduced.

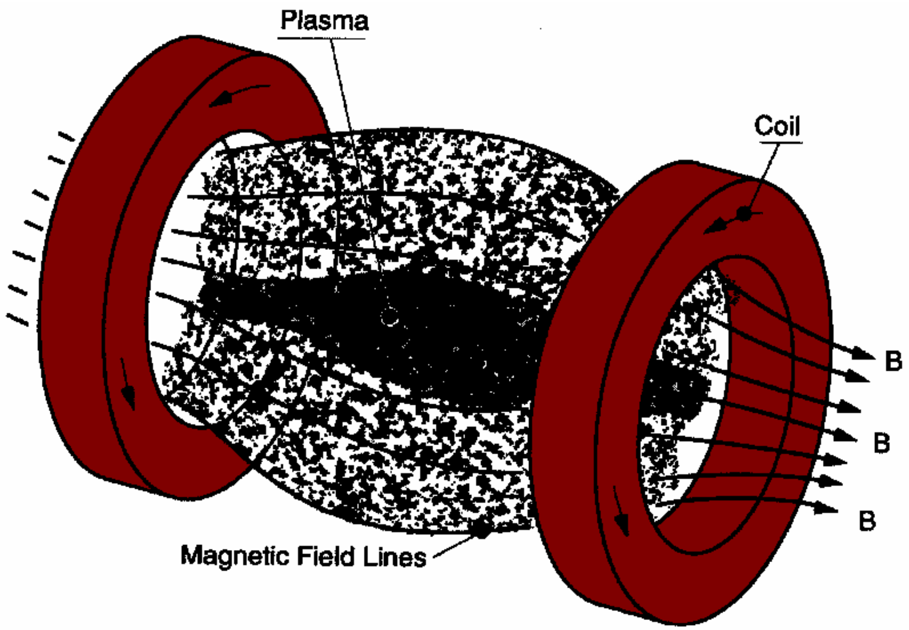
On the other hand $$\frac{dW_\perp}{dt} = \frac{d}{dt}(\mu_m B) = B\frac{d\mu_m}{dt} + \mu_m \frac{dB}{dt}$$

hence: $$\frac{d\mu_m}{dt} = 0 \quad , \quad \mu_m = const. \tag{3.24}$$

The magnetic moment is constant under a slow drift of the guiding center (slow in comparison to the orbit motion).

→ **adiabatic invariant**

Example: magnetic mirror



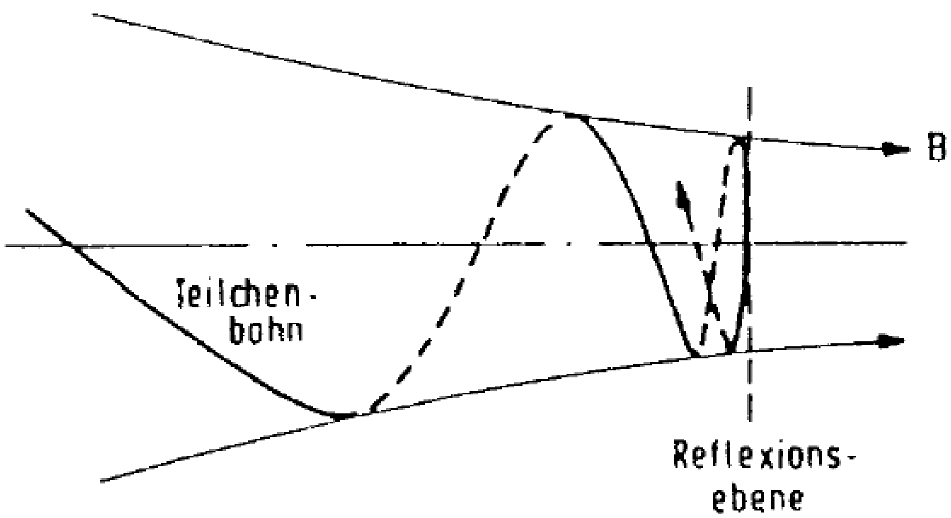
If a charged particle moves into a zone with stronger magnetic field, the kinetic energy of the gyration increases due to the invariance of the magnetic moment. As the total kinetic energy in of the particle does not change (no collisions) the kinetic energy in direction of the guiding centre motion must be reduced. In order to reduce $v_{||}$ completely

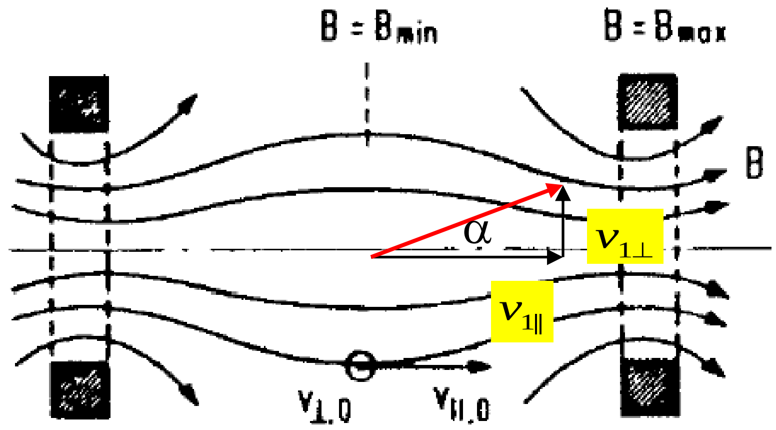
$$\mu_m (B_{\max} - B_1) = \Delta W_\perp = W_{2\perp} - W_{1\perp} > W_{\|,1}$$

The particle starts at position 1 between the coils. At Pos. 2 is the maximum of the magnetic field B_{max}.

$$\frac{B_2}{B_1} = \frac{W_{2\perp}}{W_{1\perp}} = 1 + \frac{\Delta W_\perp}{W_{1\perp}} \quad \Rightarrow \quad \frac{B_2}{B_1} - 1 = \frac{\Delta W_\perp}{W_{1\perp}} \geq \frac{W_{\parallel,1}}{W_{1\perp}} = \left(\frac{v_{1\parallel}}{v_{1\perp}}\right)^2 = (\cot\alpha)^2$$

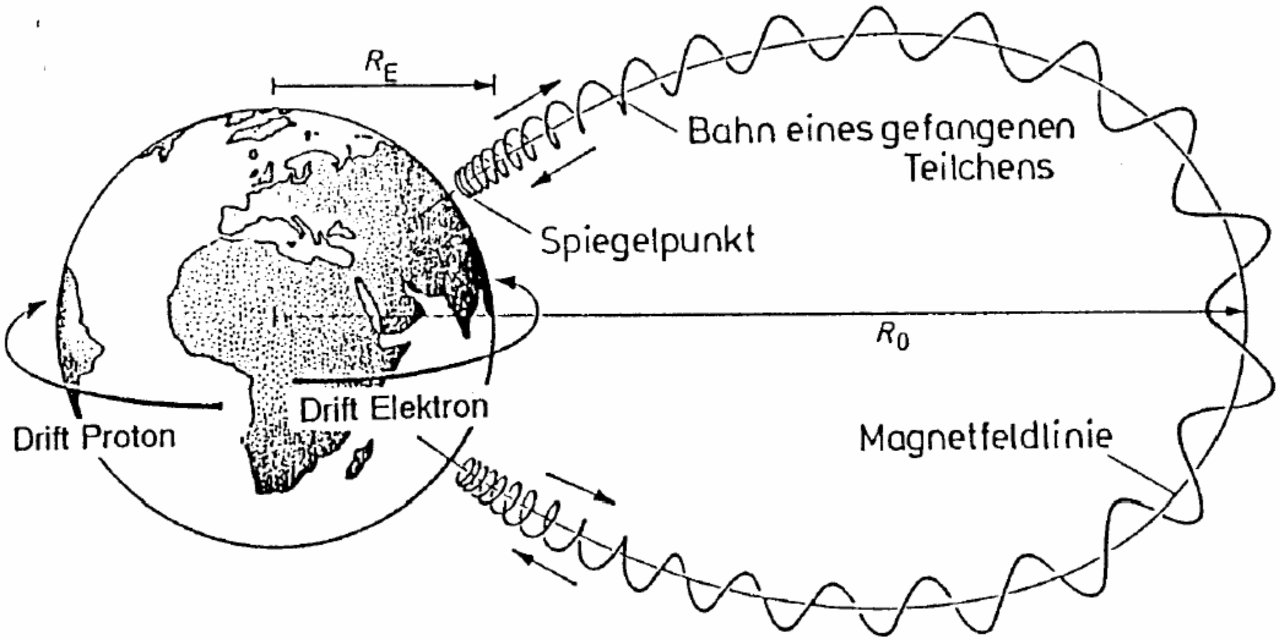
B_2/B_1 is called mirror ratio.





The loss cone is characterized by $$\alpha = \arcsin\left(\sqrt{\frac{B_{min}}{B_{max}}}\right)$$ (3.25)

This equation demonstrates that a magnetic mirror is not perfect. Particles with small velocity components perpendicular to the magnetic field are not reflected, but slowed down (as their angles towards the axis are smaller then α).



Particle motion within the earth magnetic field. At the polar regions magnetic mirrors exist and the curvature drift lead to an equatorial current.

Magnetic confinement is used in the field of ion source in

- Electron Cyclotron Resonance Ion Source (ECRIS) (resonance heating of plasma using rf)
- Confinement of ions in an Electron Beam Ion Source (EBIS)
- Multicusp-Ion Sources
- Motion of ions in an EBIS or a Penning trap