# Radio Frequency Quadrupole

- Acceleration with static $E$-field: maximum beam energy limited by maximum dc-voltage that can be realized within reasonable dimensions. Limit is at some 10 MV.
- Dimensions can be decreased by using fields that oscillate in time $E(t) = E_0 cos(\omega t)$
- Disadvantage of $E(t)$:
  - just particles that arrive within periodically occurring "good time slots" are accelerated
  - particles arriving outside a slot are decelerated, not accelerated, too weakly accelerated, ….
  - duration of "good slot" $\Delta T$ depends on frequency $\omega$: $\Delta T \cdot \omega \approx 60°$
  - beam durations from source/LEBT are much longer w.r.t. slot duration
  - particles outside slots will be lost
- Accelerate all particles, i.e. no losses:
  - need to arrange particles inside slots. This process is called "bunching"
  - particles within a "good slot" form the "bunch"
- Beam arranged into slots is "bunched beam"
- Bunched beam will de-bunch, i.e. exceed slot-border, if bunches are not kept together. This is due to tiny differences within individual particle velocities

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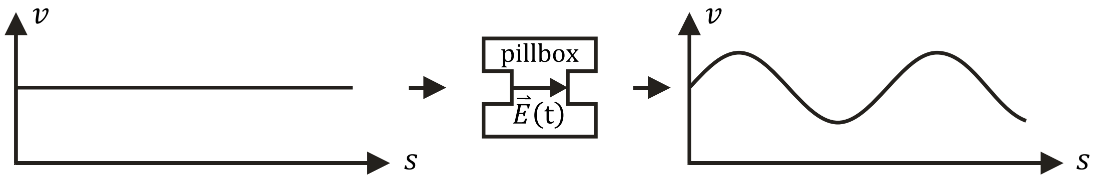
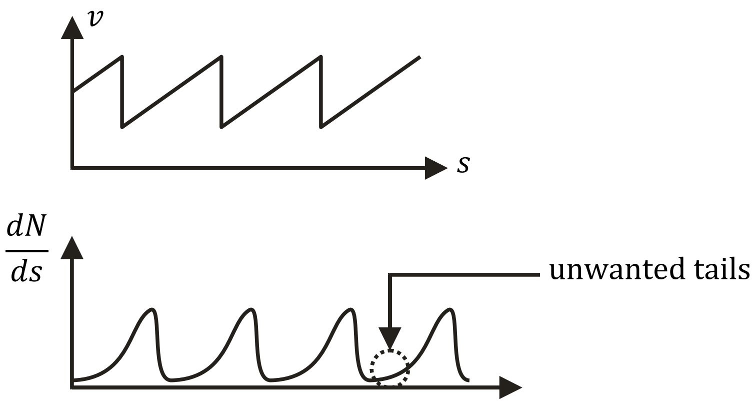
velocity modulation with pillboxes leaves "tails"

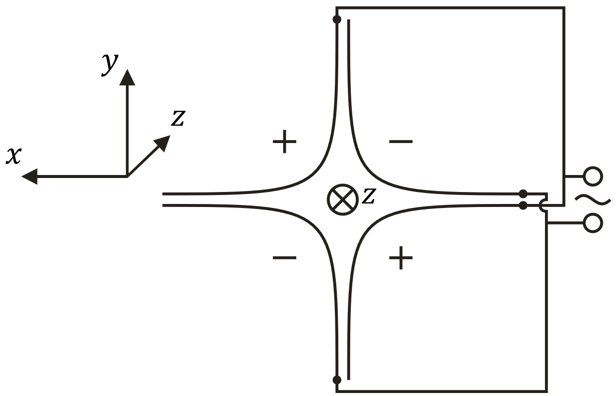



after some drift:

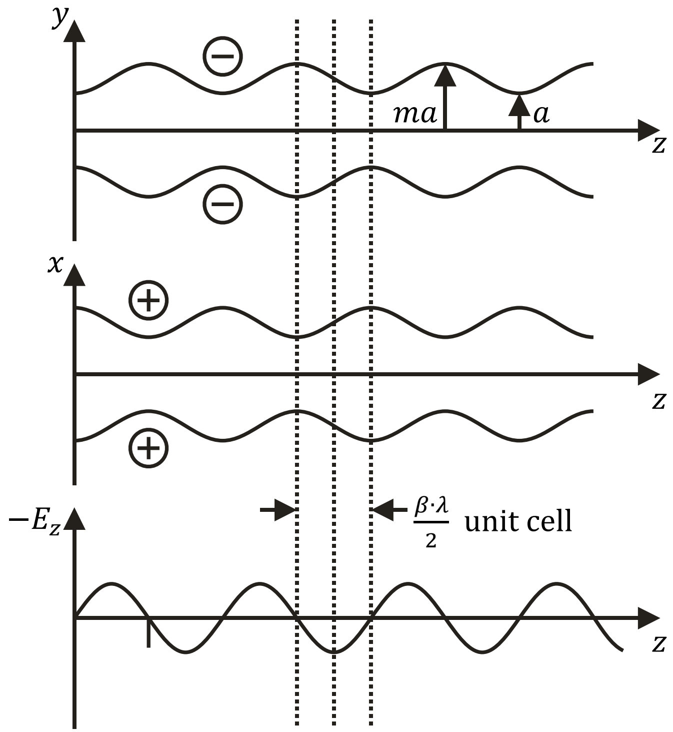



<u>Bunching is done most efficiently (hadrons !) with **R**adio **F**requency **Q**uadrupole (RFQ):</u>

<u>bunching + focusing device:</u>




4 Poles, rf-driven → RFQ




- focusing in each plane changes with time
- particles move along $z$
- focusing varies along $z$

→ net focussing like F0D0

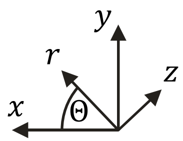
<u>description of fields:</u>

$\vec{B}$ close to beam axis $\approx 0$

$\vec{E}(r,\Theta,z,t) = -\vec{\nabla}\cdot\Psi(r,\Theta,z)\cdot\sin(\omega t+\Phi_0)$

at beam $\Delta\Psi = 0$

$$\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\Theta^2}+\frac{\partial^2\Psi}{\partial z^2}=0$$



general solution:

$$\Psi(r,\Theta,z)=\sum_{s=0}^{\infty} A_s r^{2(2s+1)} \cdot \cos[2(2s+1)\Theta] + \sum_{n=1}^{\infty}\sum_{s=0}^{\infty} A_{ns} I_{2s}(knr) \cdot \cos(2\Theta s) \cdot \sin(knz)$$

$I_1$: mod. Bessel-function

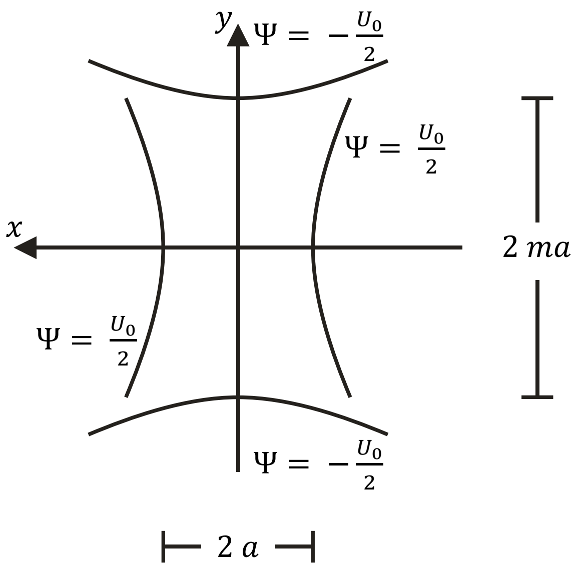
simplification/trick:

- use just terms with $s = 0$ & $n = 1$
- shape electrodes such that ($\Psi$ = const) - surface = electrode-surface
  "two term potential function"

$$\rightarrow \Psi(r,\Theta,z) = A_0 \cdot r^2 \cdot \cos 2\Theta + A_{10} \cdot I_0(\mathrm{kr}) \cos kz\,; \quad k = \frac{2\pi}{\beta_s \cdot \lambda} \qquad (*)$$

$A_0$ & $A_{10}$ from evaluation at pole-tips:

$z = 0$:




$$\Longrightarrow A_0 = \frac{U_0}{2a^2} \cdot \frac{I_0(ka)+I_0(kma)}{m^2 I_0(ka)+I_0(kma)}\,; \qquad A_{10} = \frac{U_0}{2} \cdot \frac{m^2-1}{m^2 I_0(ka)+I_0(kma)}$$

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$A_0 = \frac{U_0}{2a^2} \cdot X(a, m, \beta_s, \lambda)$ ; $A_{10} = \frac{U_0}{2} \cdot A(a, m, \beta_s, \lambda)$ (∗∗)

(∗∗) into (∗) → (Ψ = const) - surface = electrode-surface

$\vec{E} = -\vec{\nabla} \cdot \Psi$:

$$E_x = -\frac{XU_0}{a^2} \cdot x - \frac{k \cdot A \cdot U_0}{2} \cdot I_1(kr) \cdot \frac{x}{r} \cdot \cos kz$$

$$E_y = \frac{XU_0}{a^2} \cdot y - \frac{k \cdot A \cdot U_0}{2} \cdot I_1(kr) \cdot \frac{y}{r} \cdot \cos kz$$

transv. quad. transv. rf − defocusing

$$E_z = \frac{AU_0}{2} \cdot k \cdot I_0(kr) \cdot \sin kz$$

$X$ ≙ focusing efficiency

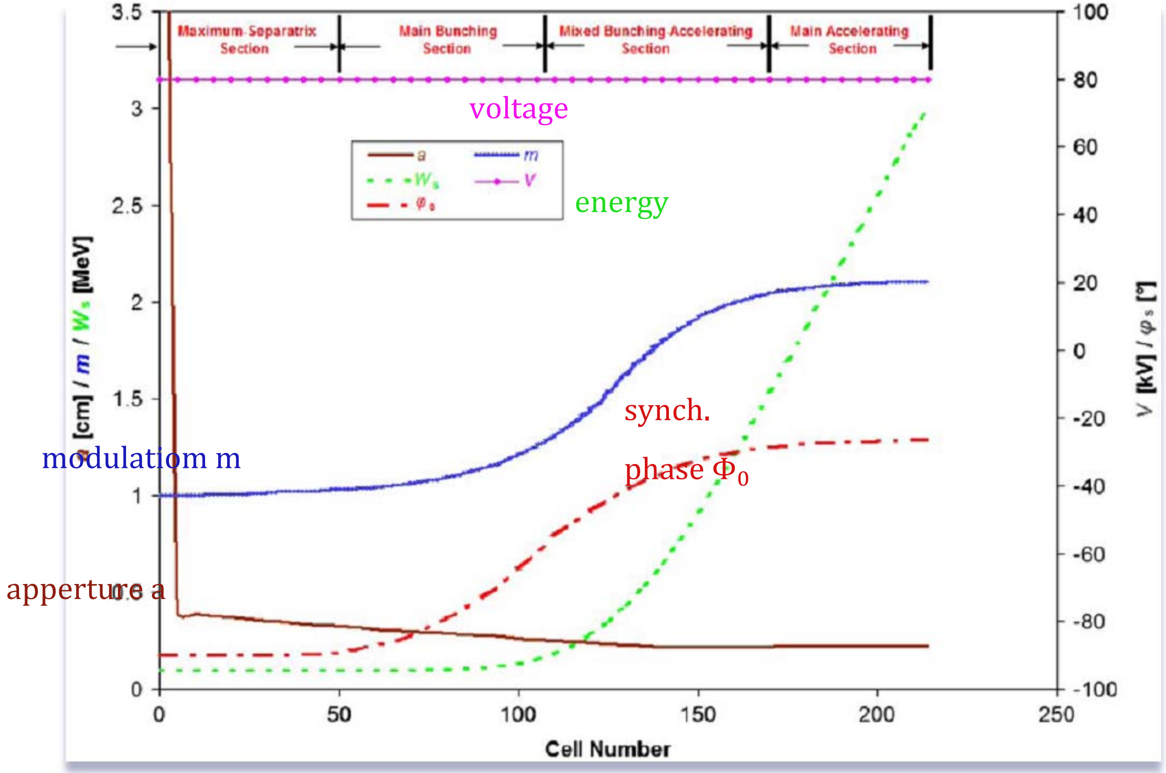
$A$ ≙ acc.eff. (= 0 for $m = 1$)

in RFQ transverse & longitudinal dynamics are coupled!

RFQ has only one knob: $U_0$

$a, m, \Phi_0$ are frozen by geometry!

RFQ Parameters vs Length




protons, length = 3.2 m, $\beta_{in}$ = 0.014, $\beta_{out}$ = 0.08, resonance frequency: 325 MHz

HLI 4-rod RFQ, GSI/Germany

Deuterium – Uranium, length: 2.055 m, $\beta_{\text{in}} = 0.0023$, $\beta_{\text{out}} = 0.025$, res. frequency: 108.408 MHz

4-rod RFQ at SARAF/Israel

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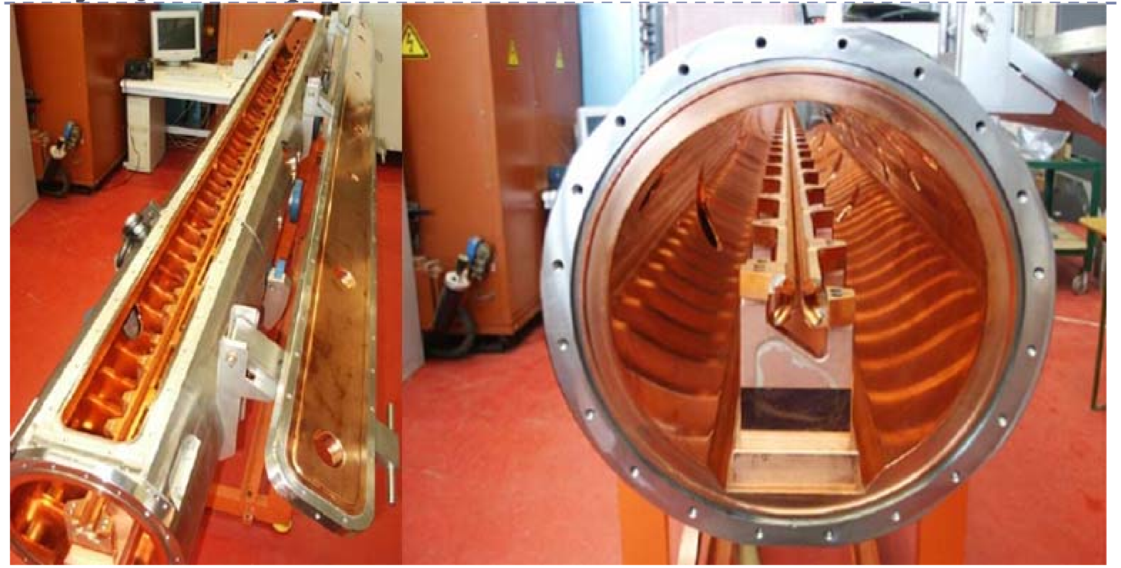
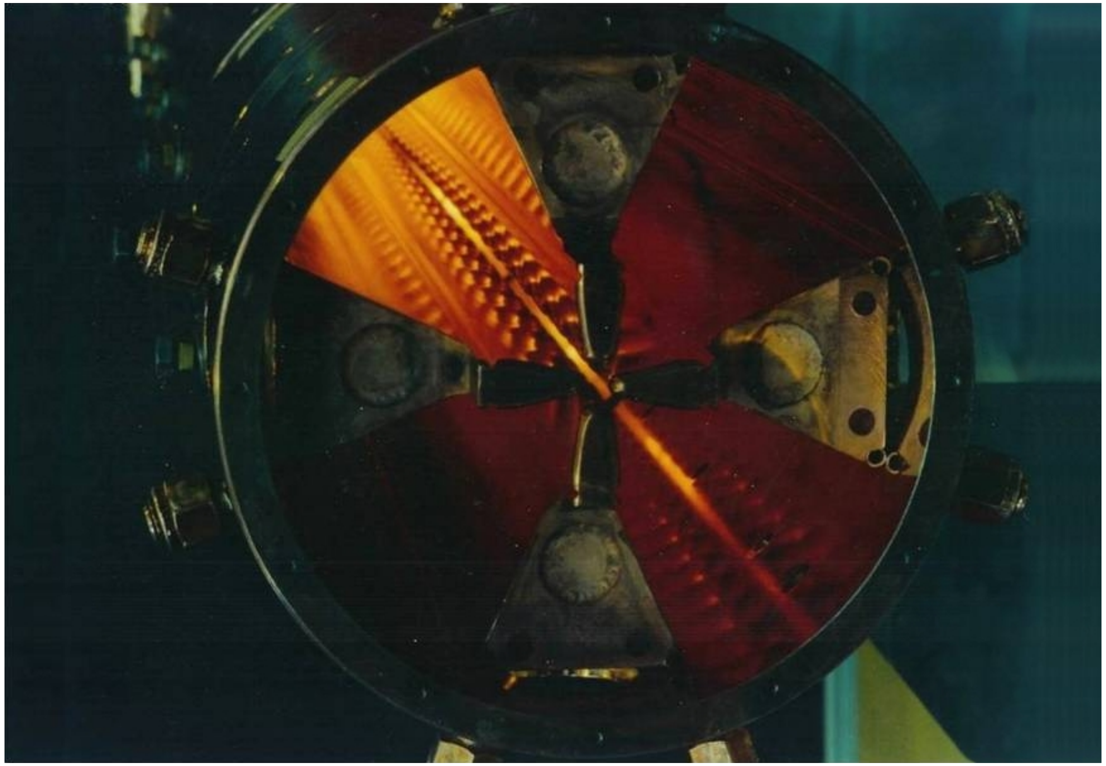


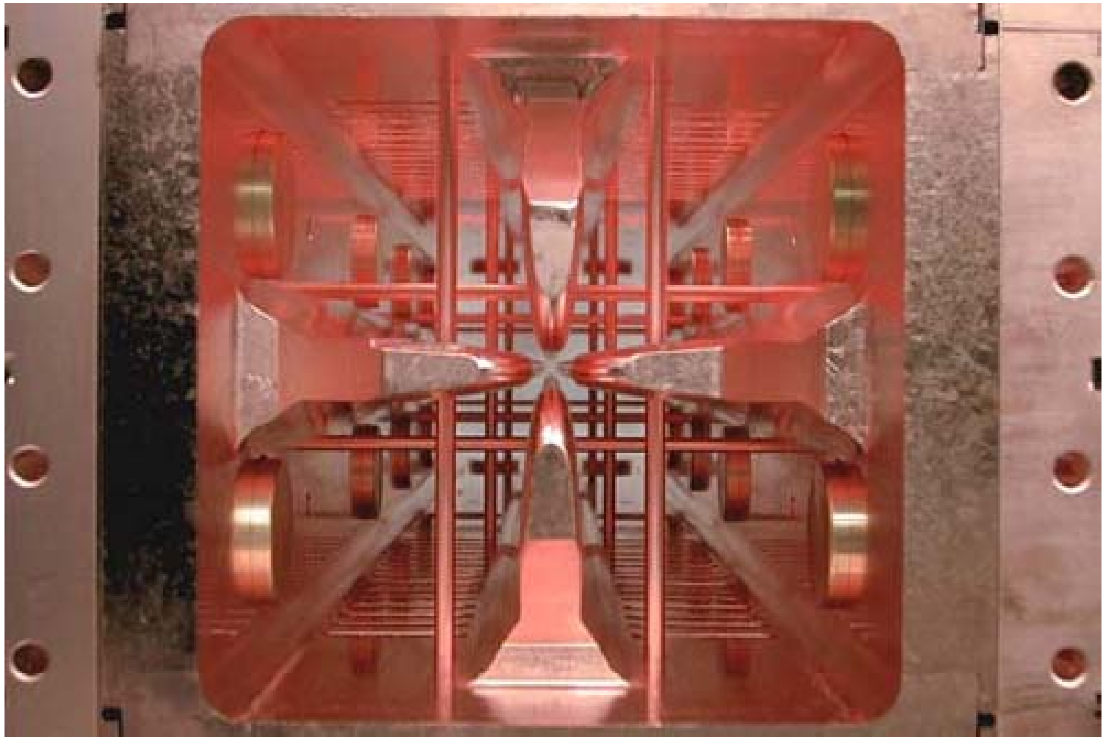
SARAF-RFQ: 3 MeV D+,  res. frequency: 175 MHz,  250 kW CW

ATS 4-vane RFQ



Protons, length: 2.89 m, $\beta_{in} = 0.015$, $\beta_{out} = 0.065$, res. frequency: 425 MHz

4-vane RFQ at SNS/Oak Ridge/USA



$H^{-}$, length: 3.7 m, $\beta_{\text{in}} = 0.012$, $\beta_{\text{out}} = 0.073$, res. frequency: 402.5 MHz