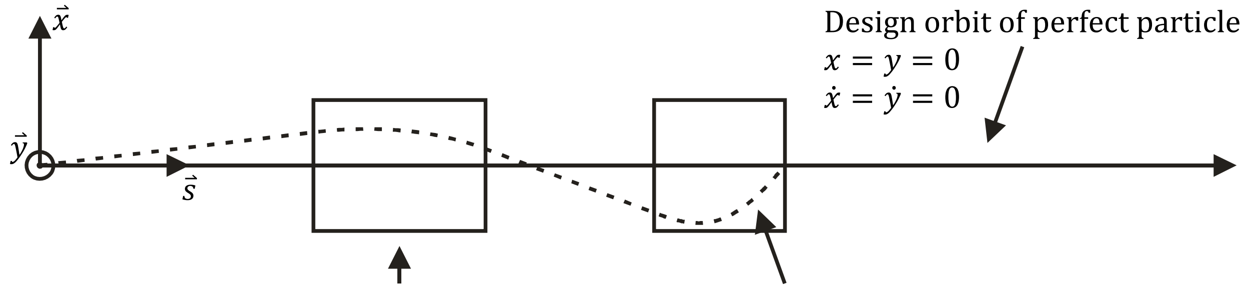
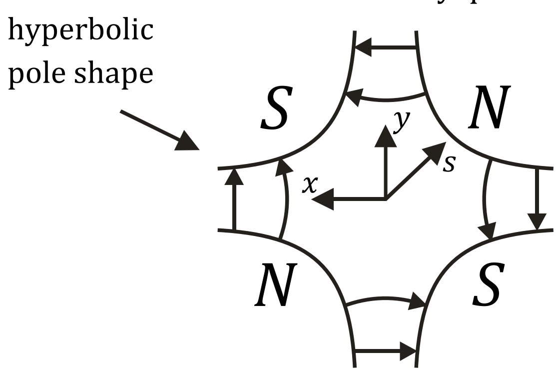
Transverse Dynamics

- transverse differential equations
- Hill's equations
- matrix formalism

Transverse Differential Equations



- correction element for real particles i.e. $x \neq 0, y \neq 0,\ \dot{x} \neq 0,\ \dot{y} \neq 0$
- correction ~ displacement in x & y
- correction by quadrupoles (4-poles)

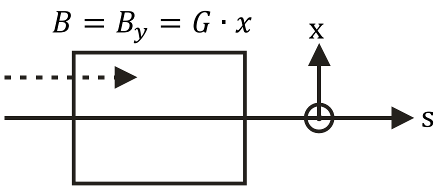


$$\vec{B} = G \cdot \begin{bmatrix} y \\ x \\ 0 \end{bmatrix} \qquad \vec{\nabla}\vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = 0$$

"Gradient $\left[\frac{\mathrm{T}}{\mathrm{m}}\right]$"

- focuses in x $(G > 0)$
- de-focuses in y $(G > 0)$
- $G < 0 \rightarrow$ vice versa

horizontal:



$F_x = -q \cdot e \cdot v \cdot B = A \cdot m_0 \gamma \cdot \ddot{x}$

$-q \cdot e \cdot v \cdot B = A \cdot m_0 \gamma \cdot v^2 x'', \quad x'' := \frac{\partial^2 x}{\partial s^2}$

$x'' = -\frac{q \cdot e \cdot B}{A \cdot m_0 \gamma \cdot v} = -\frac{q \cdot e \cdot G}{p_0} \cdot x := -kx$

$x'' = -kx$

vertical:

$y'' = +ky$ (!)

approach neglects x', since $\dot{x} \ll v$
→ paraxial approximation

Hill's Equations

$$x'' + x\left[k + \frac{1}{R^2}\right] = \frac{1}{R}\frac{\delta p}{p_0}$$

$$y'' - k \cdot y = 0$$

$k = k(s)$, $R = R(s) = \infty$ for Linacs
(curvature of design orbit)

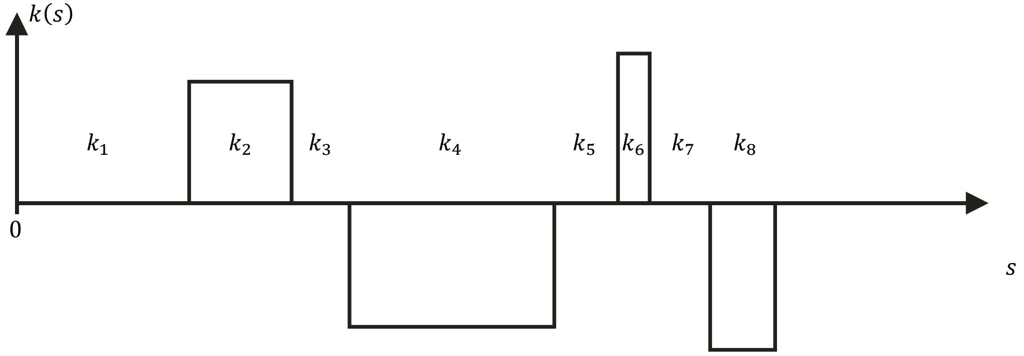
$x'' + k(s)x = 0$

$y'' - k(s)y = 0$

Matrix Formalism

$$x'' + k(s) \cdot x = 0$$

- general solution too hard
- solve piecewise, since k is from finite elements



$k = \text{const.} < 0$; $K \coloneqq |k|$

$C \coloneqq \cosh\left[\sqrt{K}s\right]$, $S \coloneqq \frac{1}{\sqrt{K}} \cdot \sinh\left[\sqrt{K}s\right]$

$x = x_0 \cdot C + x_0' \cdot S$

$x' = x_0 \cdot K \cdot S + x_0' \cdot C$

$x'' = x_0 \cdot K \cdot C + x_0' \cdot K \cdot S = Kx$

exp. increase of (x, x')

$k = \text{const.} > 0$

$C \coloneqq \cos\left[\sqrt{k}s\right]$, $S \coloneqq \frac{1}{\sqrt{K}} \cdot \sin\left[\sqrt{k}s\right]$

$x = x_0 \cdot C + x_0' \cdot S$

$x' = -x_0 \cdot k \cdot S + x_0' \cdot C$

$x'' = -x_0 \cdot k \cdot C - x_0' \cdot k \cdot S = -kx$

oscillation of (x, x')

$$\begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \begin{bmatrix} x_0 \\ x_0' \end{bmatrix}$$

$$\coloneqq R \begin{bmatrix} x_0 \\ x_0' \end{bmatrix}, \quad \det R = 1$$

S: sin-like solution

C: cos-like solution

(check!)

$$\begin{bmatrix} x \\ x' \end{bmatrix}_s = R[k_n] \cdot R[k_{n-1}] \cdots R[k_2] \cdot R[k_1] \cdot \begin{bmatrix} x_0 \\ x_0' \end{bmatrix}$$

$$= \tilde{R}[k_1 \ldots k_n] \begin{bmatrix} x_0 \\ x_0' \end{bmatrix}, \qquad \det \tilde{R} = \det[k_1] \cdot \det[k_2] \cdots \det[k_{n-1}] \cdot \det[k_n]$$

$$= \quad 1 \quad \cdot \quad 1 \quad \cdots \quad 1 \quad \cdot \quad 1 \quad = \quad 1$$

Transport Matrices:

Drift, i.e. $\quad k \equiv 0 \qquad \begin{matrix} x'' = 0 \\ y'' = 0 \end{matrix} \qquad \begin{matrix} C = 1, & C' = 0 \\ S = s, & S' = 1 \end{matrix} \qquad R_{Dx} = R_{Dy} = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$

$$R_D = \begin{bmatrix} [R_{Dx}] & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & [R_{Dy}] \end{bmatrix} = \begin{bmatrix} 1 & s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & s \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

quadrupole: $\quad B_x = G \cdot y = \frac{p \cdot k}{q \cdot e} \cdot y\,, \; B_y = G \cdot x = \frac{p \cdot k}{q \cdot e} \cdot x$

$$x'' + kx = 0, \qquad y'' - ky = 0, \qquad \Omega := \sqrt{|k|} \cdot s$$

$$R_q(k > 0) = \begin{bmatrix} \cos\Omega & \frac{1}{\sqrt{|k|}} \sin\Omega & 0 & 0 \\ -\sqrt{|k|} \sin\Omega & \cos\Omega & 0 & 0 \\ 0 & 0 & \cosh\Omega & \frac{1}{\sqrt{|k|}} \sinh\Omega \\ 0 & 0 & \sqrt{|k|} \sinh\Omega & \cosh\Omega \end{bmatrix} \quad \begin{matrix} (x, x') \text{ oscillation} \\ (y, y') \text{ exp.growth} \end{matrix}$$

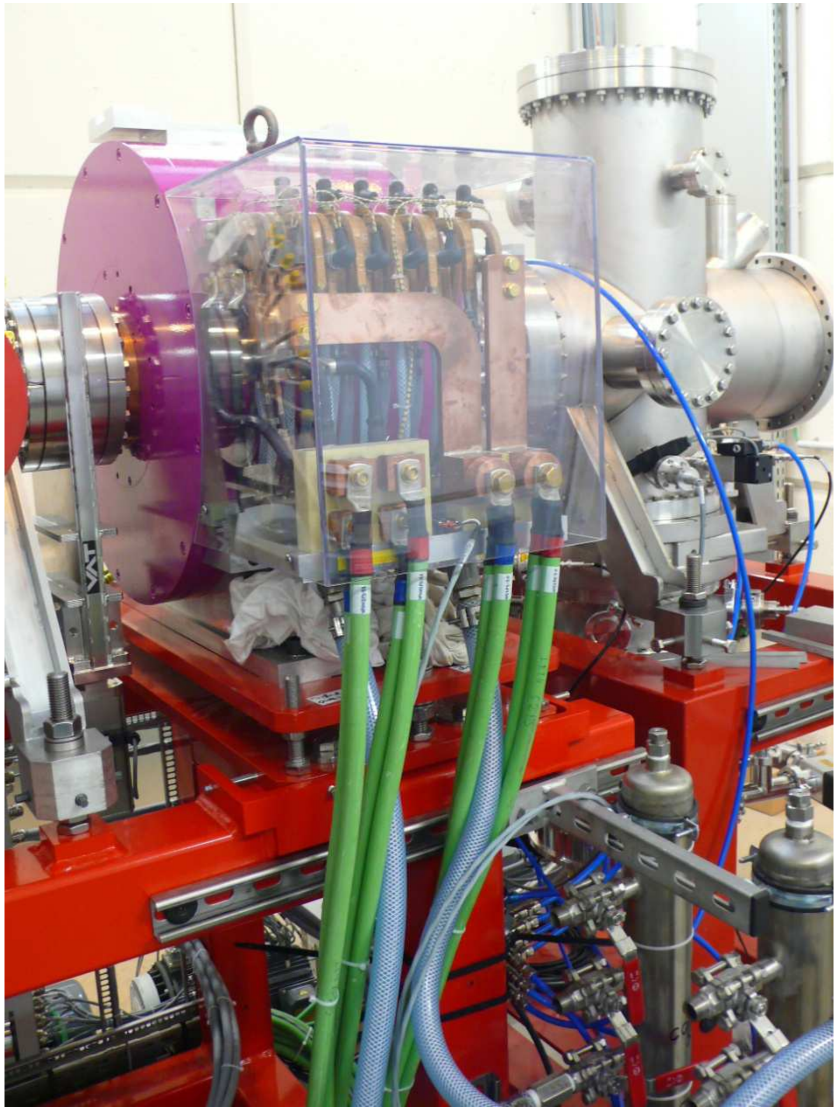
$$R_q(k<0)=\begin{bmatrix}\cosh\Omega & \frac{1}{\sqrt{|k|}}\sinh\Omega & 0 & 0\\ \sqrt{|k|}\sinh\Omega & \cosh\Omega & 0 & 0\\ 0 & 0 & \cos\Omega & \frac{1}{\sqrt{|k|}}\sin\Omega\\ 0 & 0 & -\sqrt{|k|}\sin\Omega & \cos\Omega\end{bmatrix}$$

(x, x') exp. growth

(y, y') oscillation

element of length L: $s \rightarrow L$

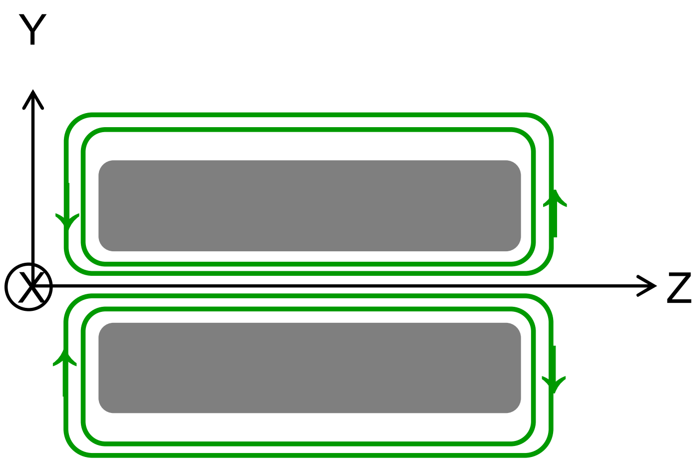
Beam Transport through Solenoids



- Introduction, definitions
- Linear transport matrix of solenoid
    - Splitting into sequences
    - Determination of images of unit vectors
- Approximation for weak fields

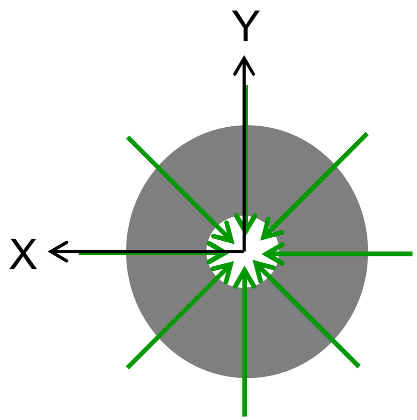
Introduction

Solenoid side view :



B - Field

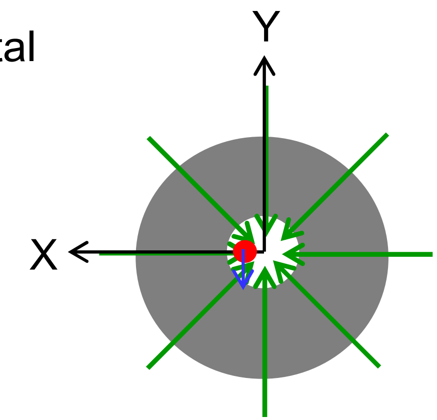
Solenoid entrance seen by beam moving towards positive z :



- coordinate system is right hand sided
- field is static and has radial symmetry
- $\vec{\nabla}\vec{B} = \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} = 0$

4D-Description Required

- a particle that moves parallel to the beam axis but with a horizontal displacement, will receive a vertical deflection
- horizontal displacement → vertical deflection
- the solenoid couples horizontal & vertical plane
- two 2d-descriptions, i.e. (x,x') & (y,y') separately, do not work
- 4d-description (x,x',y,y') is needed



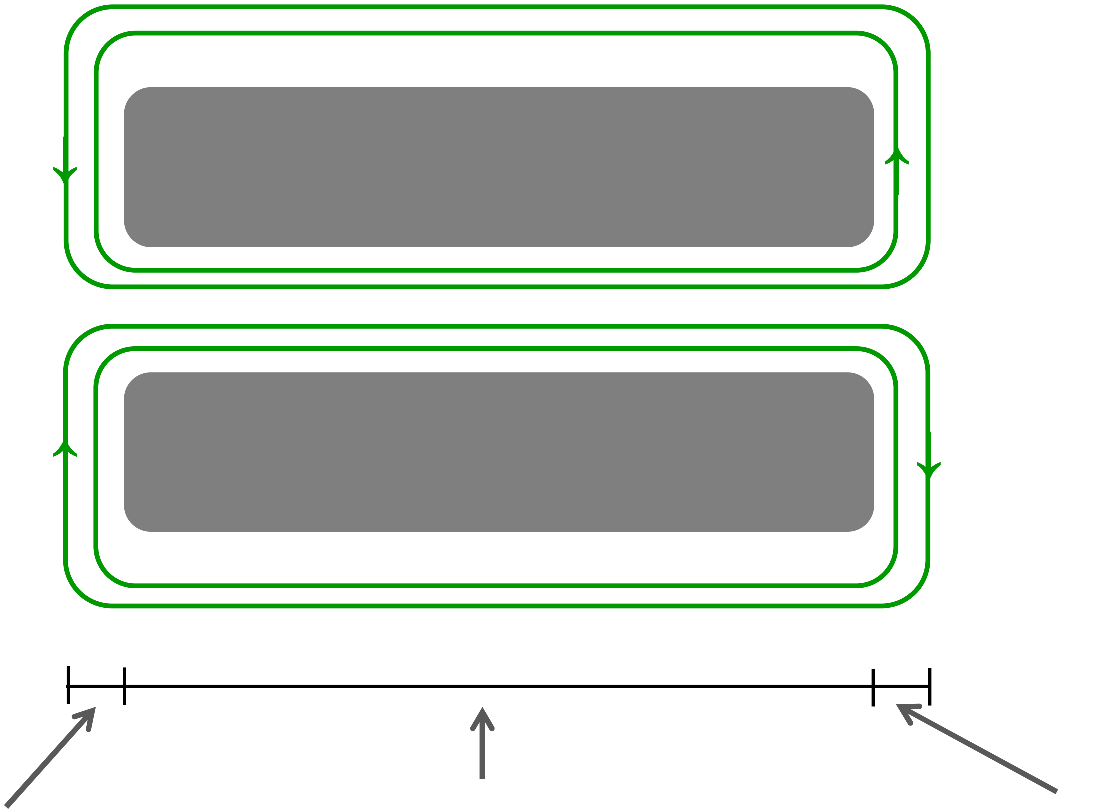
The Solenoid Transport Matrix

- in the following we just treat the linear approximation, i.e. all kicks scale to first order in displacements of x & y
- linear transport in the 4d-transverse space space using a 4x4 matrix

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_2 = M_{sol} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_1 \qquad M_{sol} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}_{sol}$$

- the task is to determine M_{sol}
- 16 matrix elements can be calculated by tracking „probe particles“

Splitting of Transport Matrix into 3 Submatrices



- entrance fringe
- radial field B_r dominates
- $M := M_{fi}$
- length = $L_f \rightarrow 0$

- main field
- B has only z-component
- $M := M_{||}$
- length = L

- exit fringe
- radial field B_r dominates
- $M := M_{fo}$
- length = $L_f \rightarrow 0$

complete solenoid matrix $M_{sol} = M_{fo} \cdot M_{||} \cdot M_{fi}$

Determining the 3 Submatrices

- Calculating M_{fi} , $M_{||}$, and M_{fo} separately
- Method: the columns of the Ms are the images of the Ms of the 4 unit vectors
- Calculation of the 4 final particle coordinates (images) of the 4 „unit (probe) particles
  “

1. $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$ 2. $\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$ 3. $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$ 4. $\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$

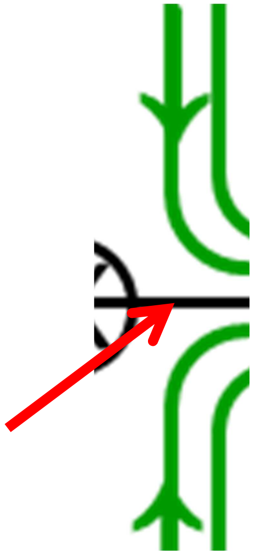
image of 1... 4

Matrix of the Entrance Fringe Field

Simplifying Assumptions:

- on axis: field is zero
- fringe field region is short
- particle does not change (x,y) inside fringe region
- particle just changes (x‘,y‘)
- for multi-poles this is the „thin lens approximation“

image of $x_1' \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$



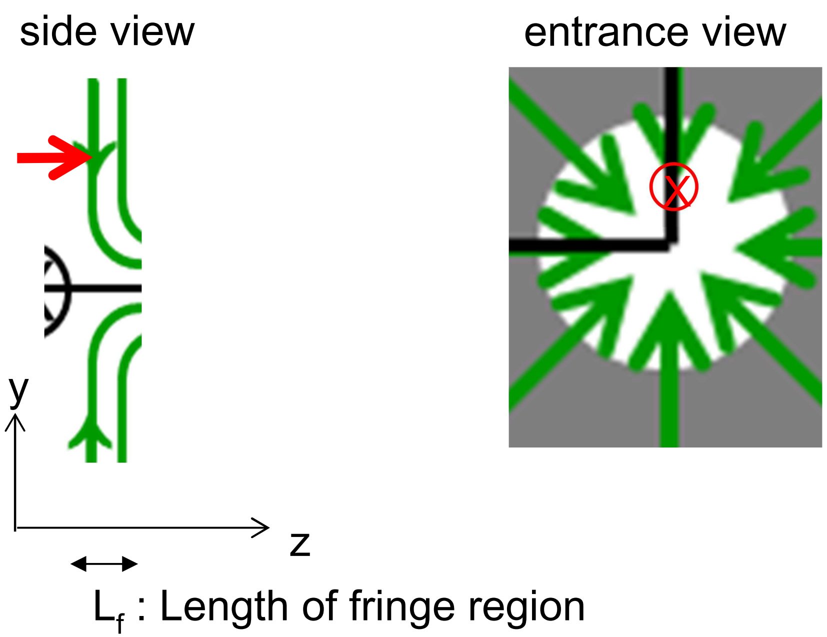
- this particle enters fringe on axis
- on axis there is no field at all
- particle coordinates remain constant

  → nothing happens at all

same argument holds for y'₁-particle :

images of $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ are $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ ➡ $M_{fi} = \begin{bmatrix} m_{11} & 0 & m_{13} & 0 \\ m_{21} & 1 & m_{23} & 0 \\ m_{31} & 0 & m_{33} & 0 \\ m_{41} & 0 & m_{43} & 1 \end{bmatrix}$

image of y $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$

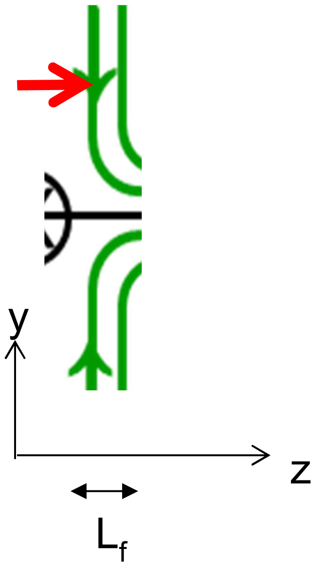


tiny local dipolar deflection

$$\delta x' = -B_y(y,z)\delta z \cdot \frac{qe}{Am\beta c} := -\frac{B_y(y,z)\delta z}{(B\rho)}$$

integrated deflection over fringe region

$$\Delta x' = \frac{-1}{(B\rho)}\int_z^{z+L_f} B_y(y,\tilde{z})\delta\tilde{z} = \frac{-1}{(B\rho)}\int_z^{z+L_f}\int_0^y \frac{\delta B_y(\tilde{y},\tilde{z})}{\delta\tilde{y}}\delta\tilde{y}\delta\tilde{z}$$



integrated deflection over fringe region

$$\Delta x' = \frac{-1}{(B\rho)} \int_z^{z+L_f} B_y(y,\tilde{z})\delta z = \frac{-1}{(B\rho)} \int_z^{z+L_f} \int_0^y \frac{\delta B_y(\tilde{y},\tilde{z})}{\delta \tilde{y}} \delta\tilde{y}\delta\tilde{z}$$

use a trick in exploiting solenoid B-field properties

$$\vec{\nabla}\vec{B} = \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} = \frac{1}{r}\frac{\delta}{\delta r}(rB_r) + \frac{\delta B_z}{\delta z} = 0$$

cartesian cylindric

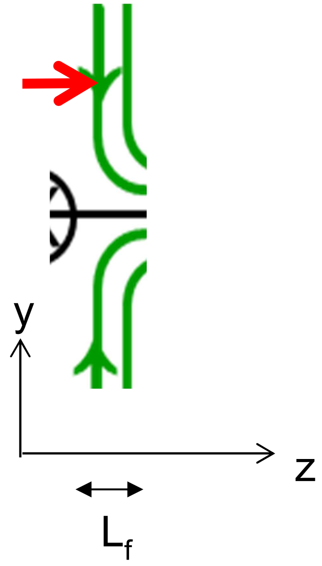
linear approximation: B_r = const • r $\quad \frac{\delta B_r}{\delta r} = \frac{B_r}{r}$

for probe particle: x = 0 → r = y $\quad \frac{\delta B_r}{\delta r} \rightarrow \frac{\delta B_y}{\delta y}$

$$\frac{\delta B_y}{\delta y} = -\frac{1}{2}\frac{\delta B_z}{\delta z} \quad \Longrightarrow \quad \Delta x' = \frac{1}{2(B\rho)} \int_z^{z+L_f} \int_0^y \frac{\delta B_z(\tilde{y},\tilde{z})}{\delta \tilde{z}} \delta\tilde{y}\delta\tilde{z}$$

„divergence = 0 trick“ eliminates transv. B-components !!

$$\Delta x' = \frac{1}{2(B\rho)} \int_z^{z+L_f} \int_0^y \frac{\delta B_z(\tilde{y},\tilde{z})}{\delta \tilde{z}} \delta\tilde{y}\delta\tilde{z}$$



y-integration is trivial :

$$\Delta x' = \frac{y}{2(B\rho)} \int_z^{z+L_f} \frac{\delta B_z(\tilde{y},\tilde{z})}{\delta \tilde{z}} \delta\tilde{z} = \frac{y \cdot B_z}{2(B\rho)} \equiv \frac{y \cdot B}{2(B\rho)}$$

1. B_z @ beginning of z-integration path is zero
2. B_z @ end of z-integration path is design solenoid field B
3. 1+2 hold for any y-value

→ z-slope of B_z, i.e. dB_z/d_z does not depend on y

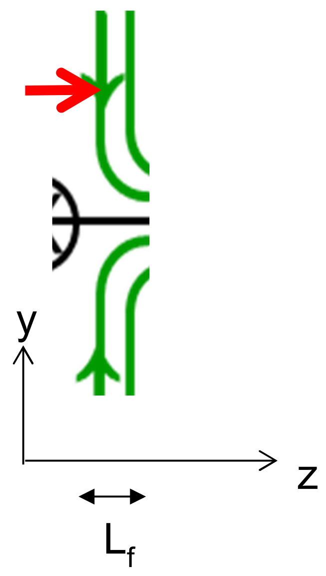
Matrices of the Fringe Fields

$$\Delta x' = \frac{y \cdot B}{2(B\rho)}$$

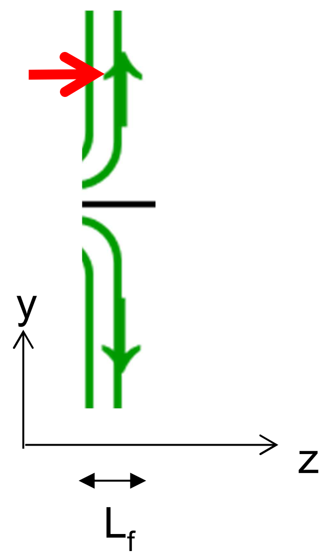
summary :

image of $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$ is $\begin{bmatrix}0\\\frac{B}{2(B\rho)}\\1\\0\end{bmatrix}$

entrance fringe :



exit fringe :



analogue :

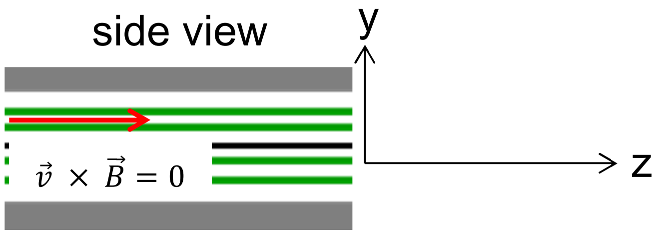
image of $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$ is $\begin{bmatrix}1\\0\\0\\\frac{B}{-2(B\rho)}\end{bmatrix}$

$$\kappa := \frac{B}{2(B\rho)}$$

$$M_{fi} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & \kappa & 0\\0 & 0 & 1 & 0\\-\kappa & 0 & 0 & 1\end{bmatrix}$$

$$M_{fo} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & -\kappa & 0\\0 & 0 & 1 & 0\\\kappa & 0 & 0 & 1\end{bmatrix}$$

Matrix of the Main Field



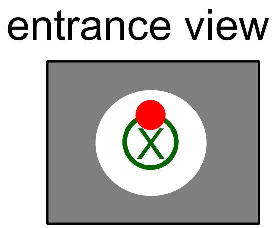
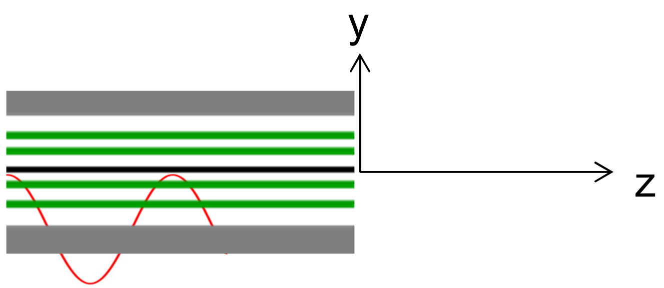


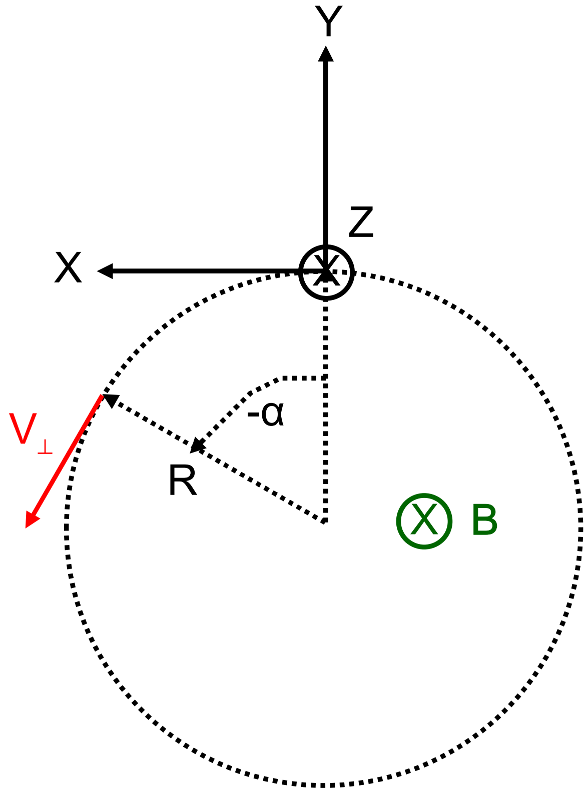
image of $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$ is $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$

image of $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$ is $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$

$$M_{||} = \begin{bmatrix}1 & m_{12} & 0 & m_{14}\\0 & m_{22} & 0 & m_{24}\\0 & m_{32} & 1 & m_{34}\\0 & m_{42} & 0 & m_{44}\end{bmatrix}$$

image of $x_1' \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$





particle rotates on arc in the x,y–plane with const R & ω

$R = \frac{Amv_\perp}{qeB}, R, v_\perp > 0 \qquad \dot{\alpha} = -\frac{v_\perp}{R}$

$$R = \frac{Am\beta c x_1'}{qeB} = \frac{(B\rho)}{B} \cdot x_1'$$

$\alpha(L) = -\frac{qeBL}{Am\beta c} = -\frac{BL}{(B\rho)} = -2\kappa L$

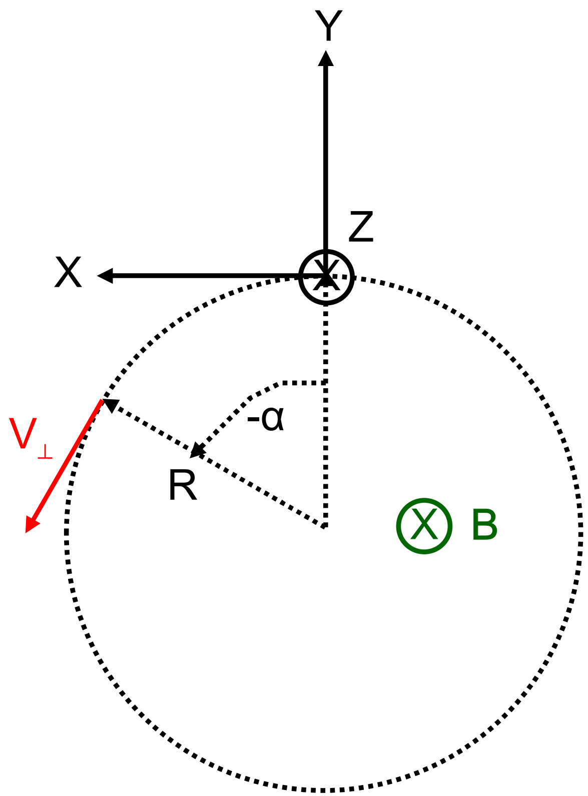
velocity rotates

$v_{x2} = v_{x1} \cdot \cos(\alpha) = \beta c \cdot x_2'$

$\beta c \cdot x_2' = \beta c \cdot x_1' \cdot \cos(\alpha)$

$x_2' = x_1' \cdot \cos(\alpha) \qquad y_2' = x_1' \cdot \sin(\alpha)$

$x_2' = x_1' \cdot \cos(\alpha) \qquad y_2' = x_1' \cdot \sin(\alpha)$



position rotates

$x_2 = -R \cdot \sin(\alpha)$

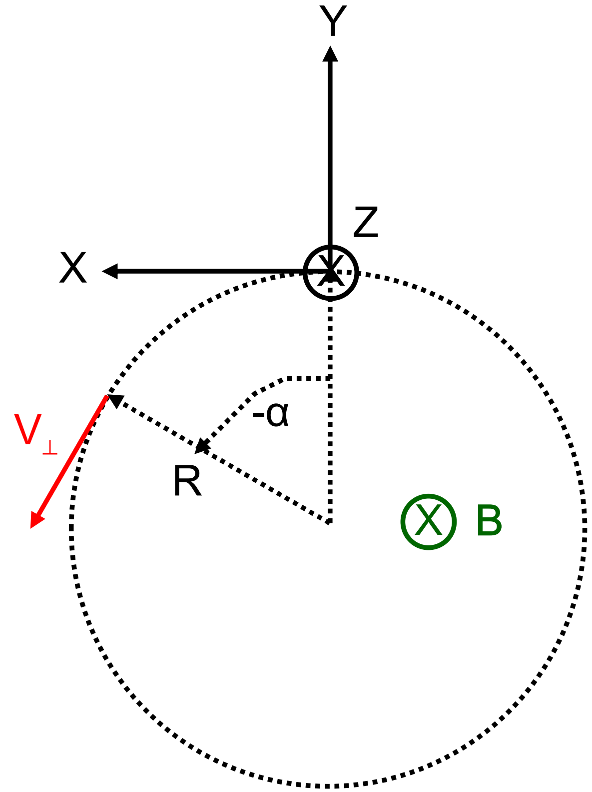
$y_2 = -R + R \cdot \cos(\alpha) = R(\cos(\alpha) - 1)$

image of $x_1' \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ is $\begin{bmatrix} -R\sin(\alpha) \\ x_1'\cos(\alpha) \\ R(\cos(\alpha)-1) \\ x_1'\sin(\alpha) \end{bmatrix} = x_1' \begin{bmatrix} -\frac{1}{2\kappa}\sin(\alpha) \\ \cos(\alpha) \\ \frac{1}{2\kappa}(\cos(\alpha)-1) \\ \sin(\alpha) \end{bmatrix}$

$$M_{||} = \begin{bmatrix} 1 & -\frac{1}{2\kappa}\sin(\alpha) & 0 & m_{14} \\ 0 & \cos(\alpha) & 0 & m_{24} \\ 0 & \frac{1}{2\kappa}(\cos(\alpha)-1) & 1 & m_{34} \\ 0 & \sin(\alpha) & 0 & m_{44} \end{bmatrix}$$

$$M_{||} = \begin{bmatrix} 1 & -\frac{1}{2\kappa}\sin(\alpha) & 0 & m_{14} \\ 0 & \cos(\alpha) & 0 & m_{24} \\ 0 & \frac{1}{2\kappa}(\cos(\alpha)-1) & 1 & m_{34} \\ 0 & \sin(\alpha) & 0 & m_{44} \end{bmatrix}$$

finally missing image of $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$



$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is +90° rotation of $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

image of rotation = rotation of image

$$M_{||}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = D \cdot M_{||}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1-\cos(\alpha)}{2\kappa} \\ -\sin(\alpha) \\ -\frac{\sin(\alpha)}{2\kappa} \\ \cos(\alpha) \end{bmatrix}$$

$$D\left(\frac{\pi}{2}\right) = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad M_{||} = \begin{bmatrix} 1 & -\frac{1}{2\kappa}\sin(\alpha) & 0 & \frac{1-\cos(\alpha)}{2\kappa} \\ 0 & \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & \frac{\cos(\alpha)-1}{2\kappa} & 1 & -\frac{\sin(\alpha)}{2\kappa} \\ 0 & \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$$

Complete Solenoid Matrix

$$M_{sol} = M_{fo} \cdot M_{||} \cdot M_{fi} = \begin{bmatrix} \frac{1+\cos(\alpha)}{2} & -\frac{\sin(\alpha)}{2\kappa} & -\frac{\sin(\alpha)}{2} & \frac{1-\cos(\alpha)}{2\kappa} \\ \kappa\frac{\sin(\alpha)}{2} & \frac{1+\cos(\alpha)}{2} & \kappa\frac{\cos(\alpha)-1}{2} & -\frac{\sin(\alpha)}{2} \\ \frac{\sin(\alpha)}{2} & \frac{\cos(\alpha)-1}{2\kappa} & \frac{\cos(\alpha)+1}{2} & -\frac{\sin(\alpha)}{2\kappa} \\ \kappa\frac{1-\cos(\alpha)}{2} & \frac{\sin(\alpha)}{2} & \kappa\frac{\sin(\alpha)}{2} & \frac{1+\cos(\alpha)}{2} \end{bmatrix}$$

$det M_{sol} = 1$, i.e. preserves 4d-rms-emittance

often found in text books : $M_{sol} = \begin{bmatrix} C^2 & \frac{SC}{K} & SC & \frac{S^2}{K} \\ -KSC & C^2 & -KS^2 & CS \\ -SC & -\frac{S^2}{K} & C^2 & \frac{SC}{K} \\ KS^2 & -SC & -KSC & C^2 \end{bmatrix}$

$C := \cos(\frac{\alpha}{2})$

$S := -\sin(\frac{\alpha}{2})$ $\qquad \alpha = -2\kappa L$

$\kappa := \frac{B}{2(B\rho)}$

- all matrix elements oszillate with B
- some oszillations increase with B, others are damped with B
- different behaviour from quadrupoles

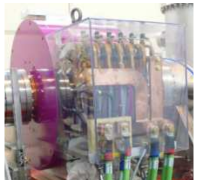
Solenoid Focusing Properties

$$\begin{bmatrix} \frac{1+\cos(\alpha)}{2} & -\frac{\sin(\alpha)}{2\kappa} & -\frac{\sin(\alpha)}{2} & \frac{1-\cos(\alpha)}{2\kappa} \\ \kappa\frac{\sin(\alpha)}{2} & \frac{1+\cos(\alpha)}{2} & \kappa\frac{\cos(\alpha)-1}{2} & -\frac{\sin(\alpha)}{2} \\ \frac{\sin(\alpha)}{2} & \frac{\cos(\alpha)-1}{2\kappa} & \frac{\cos(\alpha)+1}{2} & -\frac{\sin(\alpha)}{2\kappa} \\ \kappa\frac{1-\cos(\alpha)}{2} & \frac{\sin(\alpha)}{2} & \kappa\frac{\sin(\alpha)}{2} & \frac{1+\cos(\alpha)}{2} \end{bmatrix} \xrightarrow[\alpha = -2\kappa L \ll 1]{} \begin{bmatrix} 1 & L & \kappa L & 0 \\ -\kappa^2 L & 1 & 0 & \kappa L \\ -\kappa L & 0 & 1 & L \\ 0 & -\kappa L & -\kappa^2 L & 1 \end{bmatrix}$$

- focusing on both planes
- inter-plane coupling !!!!
- minimizing coupling for const focusing k :
  - small L → short solenoid
  - strong B → high field solenoid
  - solenoid focuses by its fringe fields NOT by its main field (as quads)
- usefull just for low beam rigidities, i.e. LEBTs
- higher rigidities → main field focusing needed → quads

Approximations and Examples

$$\begin{bmatrix} \frac{1+\cos(\alpha)}{2} & -\frac{\sin(\alpha)}{2\kappa} & -\frac{\sin(\alpha)}{2} & \frac{1-\cos(\alpha)}{2\kappa} \\ \kappa\frac{\sin(\alpha)}{2} & \frac{1+\cos(\alpha)}{2} & \kappa\frac{\cos(\alpha)-1}{2} & -\frac{\sin(\alpha)}{2} \\ \frac{\sin(\alpha)}{2} & \frac{\cos(\alpha)-1}{2\kappa} & \frac{\cos(\alpha)+1}{2} & -\frac{\sin(\alpha)}{2\kappa} \\ \kappa\frac{1-\cos(\alpha)}{2} & \frac{\sin(\alpha)}{2} & \kappa\frac{\sin(\alpha)}{2} & \frac{1+\cos(\alpha)}{2} \end{bmatrix} \xrightarrow{\alpha = -2\kappa L \ll 1} \begin{bmatrix} 1 & L & \kappa L & 0 \\ -\kappa^2 L & 1 & 0 & \kappa L \\ -\kappa L & 0 & 1 & L \\ 0 & -\kappa L & -\kappa^2 L & 1 \end{bmatrix}$$

	GSI HLI	FRANZ	FAIR p-Linac	IFMIF	SILHI
					
Ion	$^{16}O^{3+}$	p	p	d	p
Energy [keV/u]	2.5	65	95	100	95
B_{max} [T]	0.88	0.88	0.78	0.78	0.26
L [m]	0.14	0.14	0.27	0.27	0.50
α_{max} [deg]	184	192	272	133	168
K_{max} [1/m]	11.5	12.0	8.80	4.28	2.94