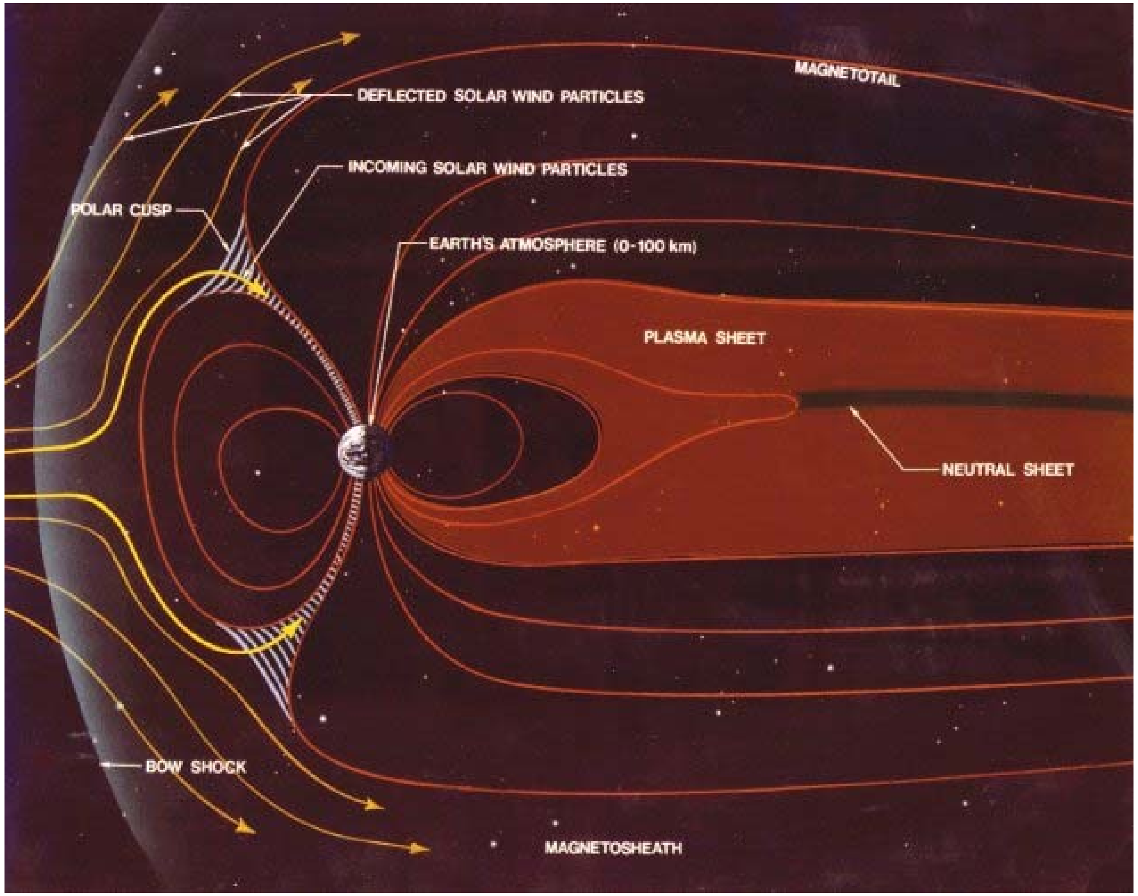
4) Magnetic confinement of plasma

Due to the shielding in the plasma, there is almost no control with electric fields. A control is possible with magnetic fields, as particles are bound to the field lines. This is called plasma confinement. It is of fundamental interest for the magnetic confinement fusion (Tokamak, Stellarator).

Example in nature: Guidance of High energetic particles of the cosmic radiation by the earth magnetic field to the poles → generation of polar lights.

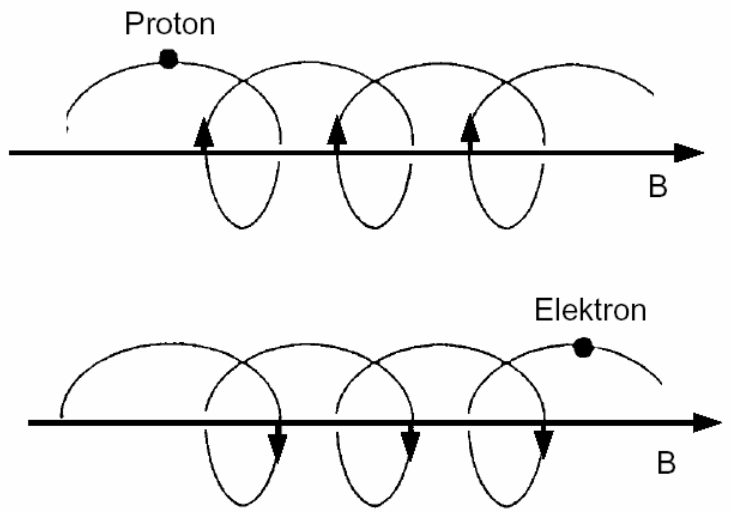


We solely take single particle motion in the magnetic field into account. If we include collective effects, we need magneto hydrodynamik (MHD) theory.

Lorentz forcet: $$m \cdot \dot{\vec{v}} = \vec{F}_L = q \cdot \vec{v} \times \vec{B}$$

As $\vec{F}_L \perp \vec{B}$ yields $m \cdot \dot{\vec{v}}_{\|} = 0$, the magnetic field has no influence on the motion parallel to the magnetic field. The direction of the velocity changes perpendicular to the magnetic field, hence the kinetic energy of the particles is constant (orbit motion).

$$m \cdot \frac{v_\perp^2}{r} = m\omega_c^2 r = q \cdot v_\perp \cdot B = q\omega_c r \cdot B$$



→ $$\omega_c = \frac{q}{m} B$$ **cyclotron frequency (4.1)**

$$\omega_{ce}[Hz] = 1.76 \cdot 10^{11} \cdot B[T] \quad , \quad \omega_{ci} = \frac{m_e}{m_i} \omega_{ce} \, ,$$

Radius on orbit: $r = \frac{m \cdot v_\perp}{q \cdot B}$

- Gyration of electrons → Plasma diagnostic
- Injection of an electromagnetic wave of $\omega = \omega_{ce}$ → plasma heating

Using $\frac{1}{2} m \cdot v_\perp^2 = kT$ (2 degrees of freedom perpendicular to B) the Lamor radius r_L is

$$r_L = \frac{m \cdot v_\perp}{q \cdot B} = \frac{\sqrt{2mkT}}{qB} = 3.4 \cdot 10^{-6} \cdot \frac{\sqrt{T_e[eV]}}{B[T]} [m] \quad (4.2)$$

Example: T_e = 1 eV, B = 1 T → r_L = 3.4 μm

Let's now investigate $m \cdot \dot{\vec{v}} = \vec{F} + q \cdot \vec{v} \times \vec{B}$

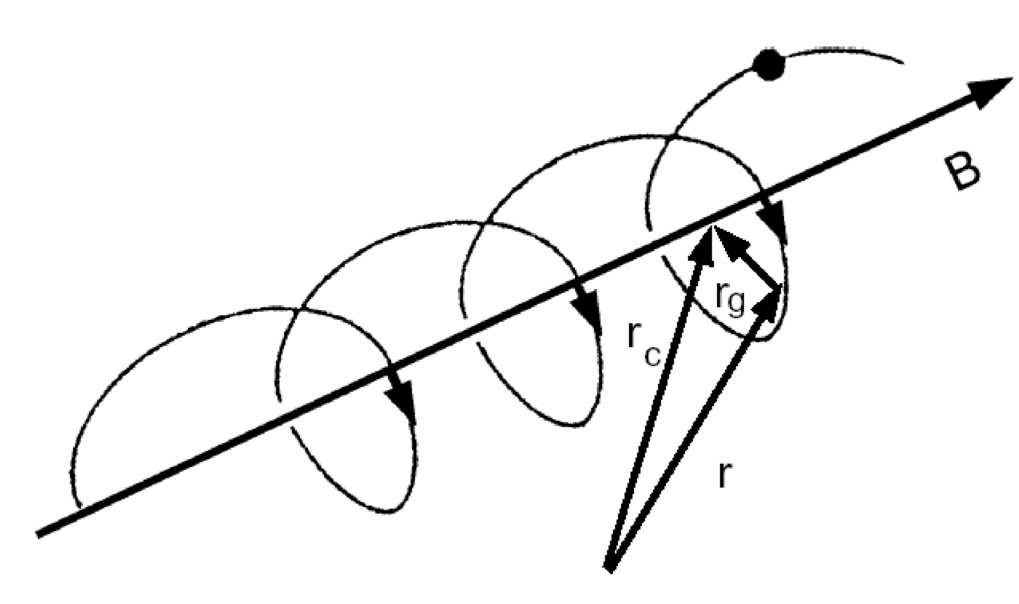
Assuming an additional force F acting on the particles ⇨ no closed orbit

For constant B und F in time and space

→ **guiding center-Ansatz**

The center motion is calculated via

$$\vec{r}_c = \vec{r} + \vec{r}_g\,, \quad \vec{r}_g = \frac{m \cdot v}{q \cdot B} \cdot \frac{\vec{v} \times \vec{B}}{v \cdot B} = \frac{m}{q \cdot B^2} \vec{v} \times \vec{B}$$



The velocity of the guiding center is give by

$$\vec{v}_c = \dot{\vec{r}}_c = \dot{\vec{r}} + \dot{\vec{r}}_g = \vec{v} + \frac{m}{q \cdot B^2} \cdot \dot{\vec{v}} \times \vec{B} = \vec{v} + \frac{1}{q \cdot B^2} (\vec{F} + q \cdot \vec{v} \times \vec{B}) \times \vec{B}$$

with $(\vec{v} \times \vec{B}) \times \vec{B} = (\vec{v} \cdot \vec{B})\vec{B} - B^2 \vec{v} = -B^2 \vec{v}_\perp$ the following result of the motion is derived

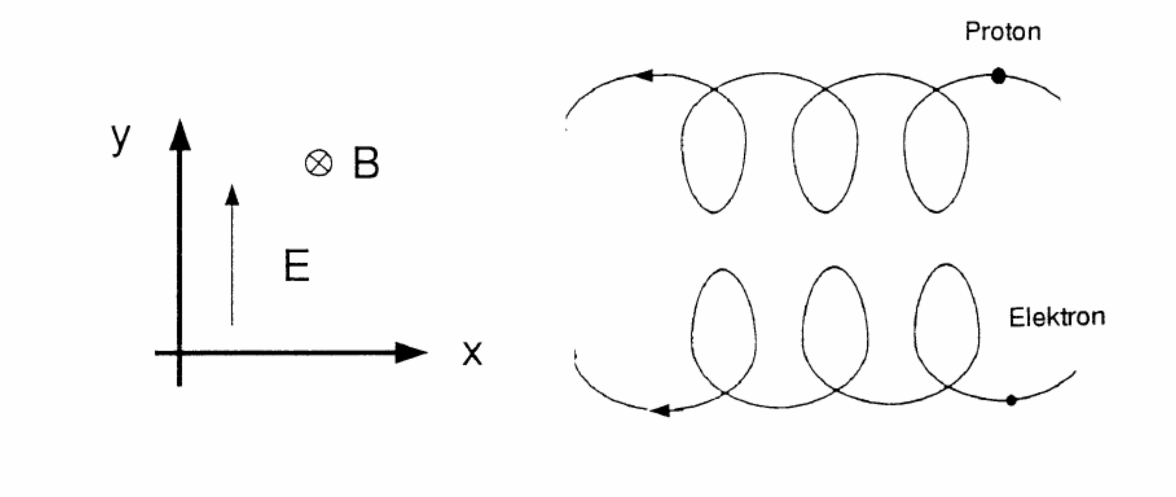
$$\vec{v}_c = \vec{v}_\parallel + \frac{\vec{F} \times \vec{B}}{q \cdot B^2} \quad \text{and} \quad m\dot{\vec{v}}_\parallel = \vec{F}_\parallel \qquad (4.3)$$

The second term is called drift velocity. This term does not accelerate and is directed prependicular to B and $F_\perp$.

Examples: **1.) elektric field** $\vec{F} = q \cdot \vec{E}$ → E x B –drift

→ $\vec{v}_D = \dfrac{\vec{E} \times \vec{B}}{B^2}$

The drift velocity is not dependent on the charge of the particle.



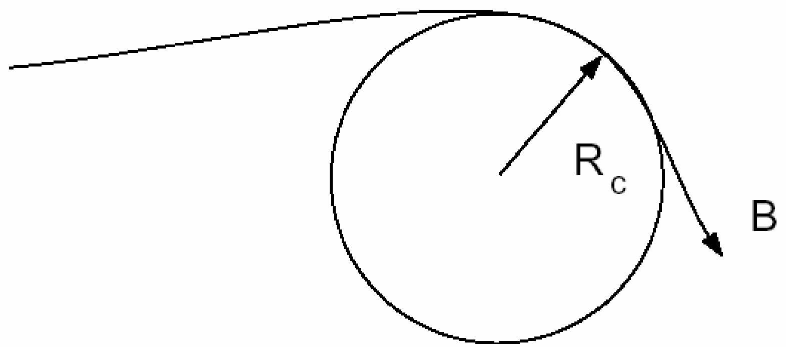
ExB-drift in direction of the x-axis. The gyration radii are not drawn to scale!

2.) Gravitation $\vec{F} = m \cdot \vec{g}$ → $\vec{v}_D = \dfrac{m \cdot \vec{g} \times \vec{B}}{q \cdot B^2}$

The particle gains energy in the direction of $\vec{g}_{\parallel}$. The drift is charge dependent and leads to a charge separation introducing the current $\vec{j}_g = n_e e \cdot (\vec{v}_i - \vec{v}_e) = n_e e \dfrac{\vec{g} \times \vec{B}}{e \cdot B^2} \left(\dfrac{m_i}{Z} + m_e \right)$

For sources this drift is negligible. More important are drifts due to gradients.

3.) $\vec{\nabla}$ **B-Drift**, if $\vec{B} = \vec{f}(x,t)$ → curvature drift



Due to $\vec{\nabla} \cdot \vec{B} = 0$ a gradient in B leads to a curvature of the field lines. This results in a

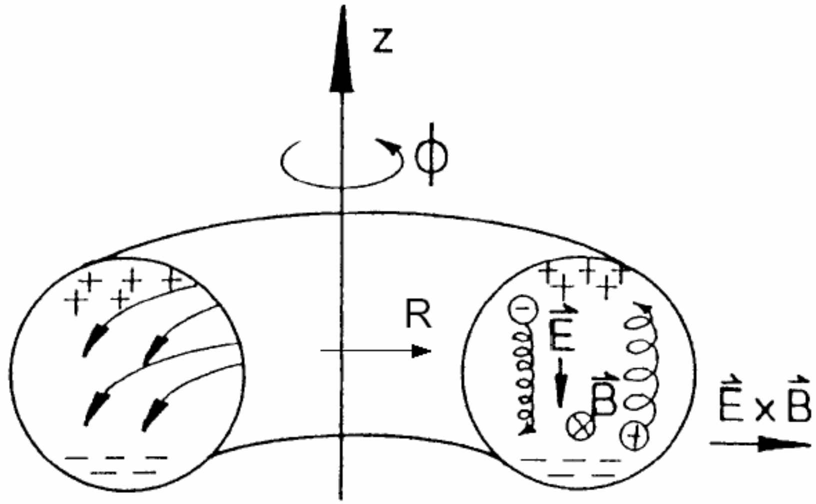
centrifugal force $\vec{F} = m \cdot \frac{\left|\vec{v}_{\|}\right|^2}{R_c} \cdot \hat{\vec{r}}_c$.

$$\vec{v}_D = \frac{m \cdot \left|\vec{v}_{\|}\right|^2}{R_c} \cdot \frac{\hat{\vec{r}}_c \times \vec{B}}{q \cdot B^2}$$

$$\frac{\hat{\vec{r}}_c}{R_c} = \frac{d\hat{\vec{T}}}{ds} = -\frac{\vec{\nabla} B}{B}$$

The centrifugal force is in direction of $-\vec{\nabla} B$ and hence

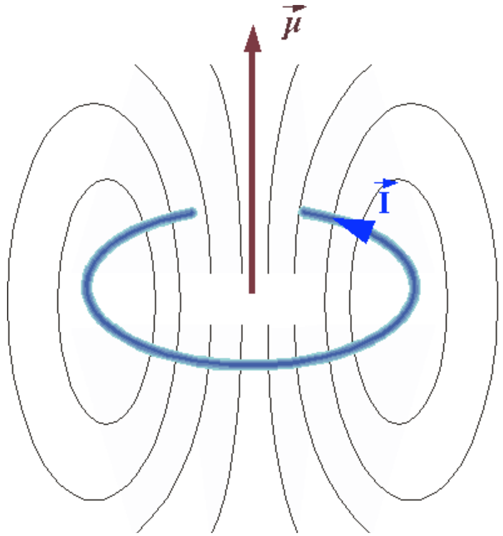
$$\vec{v}_D = -\frac{m \cdot \left|\vec{v}_{\|}\right|^2}{q \cdot B^3} \cdot \vec{\nabla} B \times \vec{B}$$



Particle drift in a toroidal magnetic field. The drift results in a charge separation and the resulting ExB-drift moves the plasma outward

Example: toroidal magnetic field

Adiabatic invariance of the magnetic moment



A charged particle on a circular orbit creates a current in the magnetic field.

$$I = q\frac{v}{2\pi \cdot r} = q\frac{\omega}{2\pi}$$

magnetic dipole moment:

$$\vec{\mu}_m = I \cdot \int_F dF \cdot \vec{n}_F = I \cdot F \cdot \vec{n}_F = \tfrac{1}{2} q \omega_c \cdot r_L^2 \cdot \vec{n}_F = \frac{q^2}{2m} B \cdot r_L^2 \cdot \vec{n}_F$$

Assuming the Lamor radius $r_L = \frac{v_\perp m}{B \cdot q}$ ➔ $\mu_m = \frac{q^2}{2m} B \cdot \left(\frac{v_\perp m}{B \cdot q} \right)^2 = \frac{m \cdot v_\perp^2}{2B} = \frac{W_\perp}{B}$ (4.4)

How does the magnetic moment change with a slow drift of the guiding centre?
Change of the magnetic flux ϕ through the orbit area!

Gyration period $\tau_C = \frac{2\pi}{\omega_c}$ and $\Delta W_\perp = q \cdot \frac{d\phi}{dt} = \pi \cdot q \cdot r_L^2 \cdot \frac{dB}{dt}$

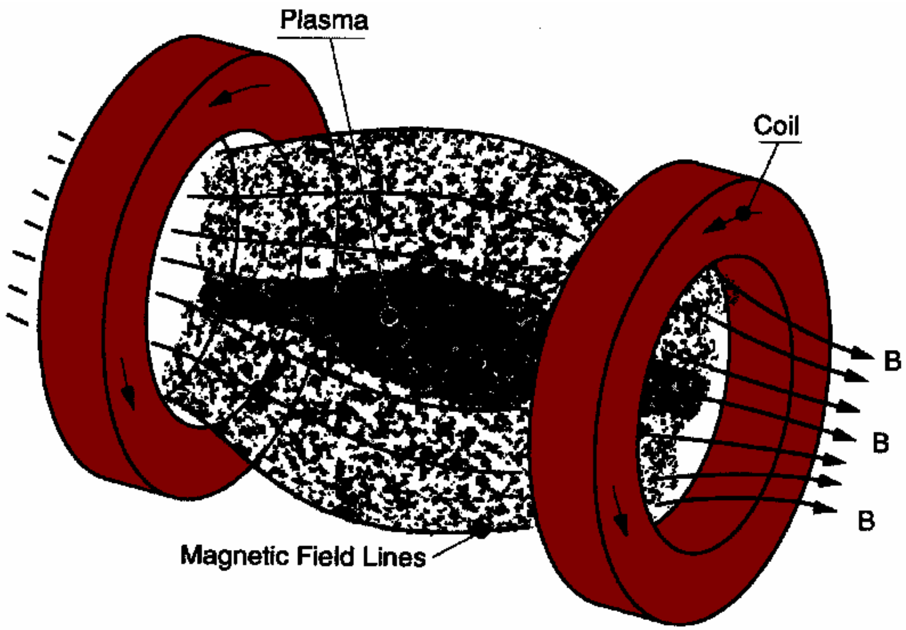
→ $\frac{dW_\perp}{dt} \approx \frac{\Delta W_\perp}{\tau_c} = \frac{\pi \cdot q \cdot r_L}{\tau_c} \cdot \frac{dB}{dt} = \mu_m \frac{dB}{dt}$ if τ_c and r_L are introduced.

On the other hand
$$\frac{dW_\perp}{dt} = \frac{d}{dt}(\mu_m B) = B\frac{d\mu_m}{dt} + \mu_m \frac{dB}{dt}$$

hence:
$$\frac{d\mu_m}{dt} = 0 \quad , \quad \mu_m = const.$$

The magnetic moment is constant under a slow drift of the guiding center (slow in comparison to the orbit motion).

→ **adiabatic invariant**



Example: magnetic mirror

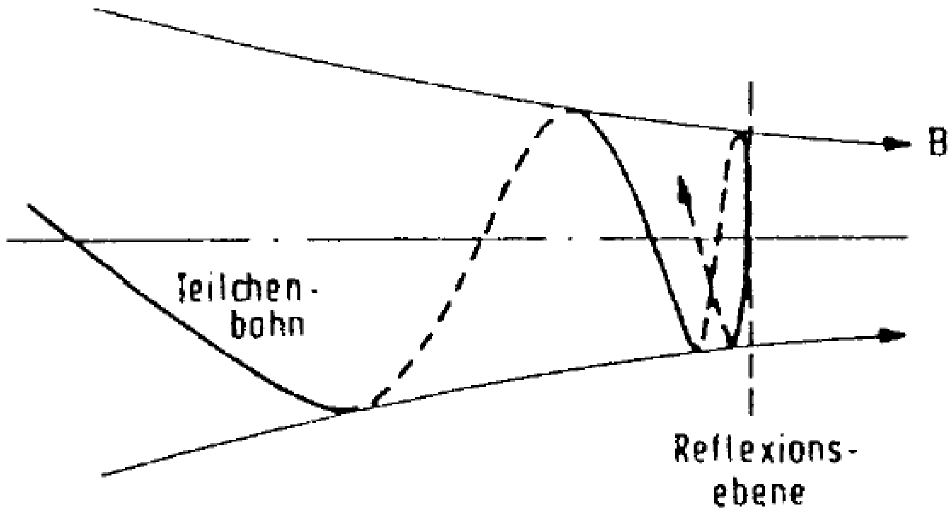
If a charged particle moves into a zone with stronger magnetic field, the kinetic energy of the gyration increases due to the invariance of the magnetic moment. As the total kinetic energy in of the particle does not change (no collisions) the kinetic energy in Direction of the guiding center motion must be reduced. In order to reduce $v_{||}$ completly

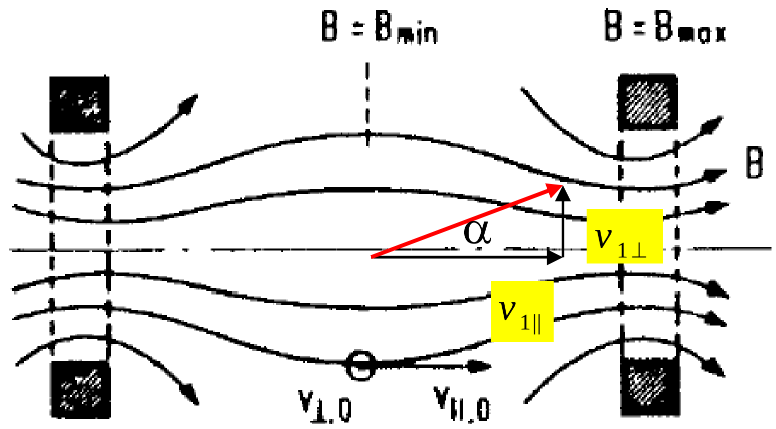
$$\mu_m (B_{\max} - B_1) = \Delta W_\perp = W_{2\perp} - W_{1\perp} > W_{||,1}$$

The particle starts at position 1 between the coils. At Pos. 2 is the maximum of the magnetic field B_{max}.

$$\frac{B_2}{B_1} = \frac{W_{2\perp}}{W_{1\perp}} = 1 + \frac{\Delta W_\perp}{W_{1\perp}} \quad \Rightarrow \quad \frac{B_2}{B_1} - 1 = \frac{\Delta W_\perp}{W_{1\perp}} \geq \frac{W_{\parallel,1}}{W_{1\perp}} = \left(\frac{v_{1\parallel}}{v_{1\perp}}\right)^2 = (\cot\alpha)^2$$

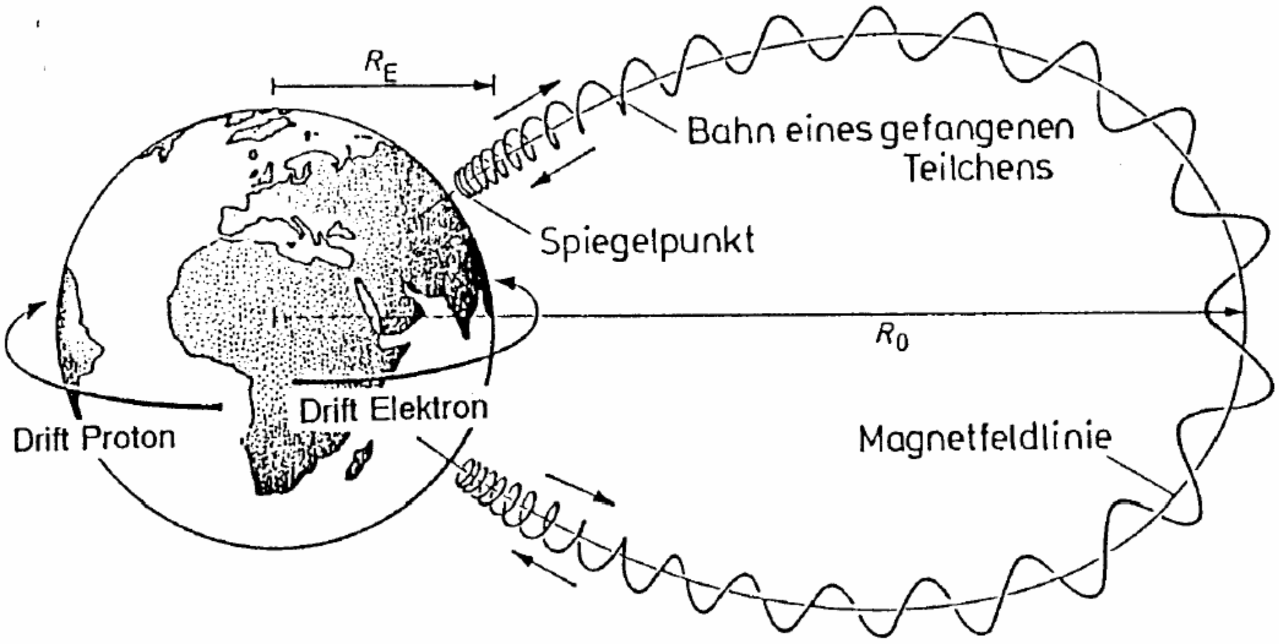
B_2/B_1 is called mirror ratio.





The loss con eis characterized by $\alpha = \arcsin(\sqrt{\frac{B_{min}}{B_{max}}})$ (4.5)

This equation demonstrates that a magnetic mirror is not perfect. Particles with small velocity components perpendicular to the magnetic field are not reflected, but slowed down (as their angles towards the axis are smaller then α).



Particle motion within the earth magnetic field. At the polar regions magnetic mirrors exist and the curvature drift lead to an equatorial current.

Magnetic confinement is used in the field of ion source in

- Electron Cyklotron Resonanz Ion Source (ECRIS) (resonance heating of plasms using rf)
- Confinement of ions in an Electrone Beam Ion Source (EBIS)
- Multicusp-Ion Sources
- Motion of ions in an EBIS or a Penning trap