Ringbeschleuniger und Speicherringe
Übungsblatt 6
Lösungen

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1 Geschwindigkeit und Impuls

Beweise:

$$\frac{dv}{v} = \frac{1}{\gamma^2}\frac{dp}{p}$$

Die Lösung ergibt sich aus:

$$\frac{d\ p}{d\ v} = \frac{m_0\ c\ d(\gamma\beta)}{c\ d\beta} = m_0\frac{d}{d\beta}\left(\frac{\beta}{\sqrt{1-\beta^2}}\right)$$

$$\rightarrow \quad \frac{d}{d\beta}\left[\beta(1-\beta^2)^{-\frac{1}{2}}\right] = (1-\beta^2)^{-\frac{1}{2}} + \beta\frac{1}{2}(1-\beta^2)^{-\frac{2}{3}}2\beta$$

$$= \frac{1-\beta^2+\beta^2}{(1-\beta^2)^{\frac{3}{2}}} = \frac{1}{(1-\beta^2)^{\frac{3}{2}}} = \gamma^3$$

$$\frac{dp}{dv} = m_0\gamma^3 \quad \rightarrow \quad \frac{dp}{m_0\ \gamma^3} = d\nu \quad \rightarrow \quad \frac{dp}{m_0 v\gamma^3} = \frac{dv}{v}$$

$$\rightarrow \quad \frac{dv}{v} = \frac{1}{\gamma^2}\frac{dp}{p}$$

2 Chromatizität

$$\frac{\Delta Q}{Q} = \xi_{rel}\frac{\Delta p}{p_0} \quad \rightarrow \quad \Delta Q = Q\cdot\xi_{rel}\cdot\delta = 27,5\cdot(-1,35)\cdot(\pm 2,5\cdot 10^{-3}) = \pm 0,093$$

3 FODO Zellen

Für die beschriebene FODO Zelle ergibt sich

$$\underline{R}_{FODO} = \begin{pmatrix}1 & 0\\ -\frac{1}{2f} & 1\end{pmatrix}\begin{pmatrix}1 & d\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{1}{f} & 1\end{pmatrix}\begin{pmatrix}1 & d\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -\frac{1}{2f} & 1\end{pmatrix} = \begin{pmatrix}1-\frac{d^2}{2f^2} & 2d+\frac{d^2}{f}\\ -\frac{d}{2f^2}\left(1-\frac{d}{2f}\right) & 1-\frac{d^2}{2f^2}\end{pmatrix}$$

Phasenvorschub und Eigenellipse:

$$\cos(\mu) = (R_{11} + R_{22}) \cdot \frac{1}{2} = 1 - \frac{d^2}{2f^2} \qquad \alpha = \frac{R_{11} - R_{22}}{2\sin(\mu)} = 0$$

$$\beta = \frac{R_{12}}{sin(\mu)} = \frac{d(2+\frac{d}{f})}{\sqrt{1-cos^2(\mu)}} = \frac{d(2+\frac{d}{f})}{\sqrt{1-(1-\frac{d^2}{2f^2})^2}} = \frac{d(2+\frac{d}{f})}{\sqrt{\frac{d^2}{f^2}-\frac{d^4}{4f^4}}}$$

$$= \frac{d(2+\frac{d}{f})}{\frac{d}{2f}\sqrt{4-\frac{d^2}{f^2}}} = \frac{2f(2+\frac{d}{f})}{\sqrt{(2-\frac{d}{f})(2+\frac{d}{f})}} = \sqrt{\frac{2+\frac{d}{f}}{2-\frac{d}{f}}} \cdot 2f$$

$$\gamma = -\frac{R_{21}}{\sin(\mu)} = \frac{\frac{d}{2f^2}(1-\frac{d}{2f})}{\sqrt{1-\cos^2(\mu)}} = \frac{\frac{d}{2f^2}(1-\frac{d}{2f})}{\frac{d}{2f}\sqrt{4-\frac{d^2}{f^2}}} = \frac{(1-\frac{d}{2f})}{2f\sqrt{1-\frac{d^2}{4f^2}}}$$

$$= \frac{(1-\frac{d}{2f})}{2f\sqrt{(1+\frac{d}{2f})(1-\frac{d}{2f})}} = \sqrt{\frac{1-\frac{d}{2f}}{1+\frac{d}{2f}}} \cdot \frac{1}{2f} = \frac{1}{\beta}$$