Ringbeschleuniger und Speicherringe
Quiz
Lösungen

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1 Fehler bei nicht-relativistischer Betrachtung

$$\begin{aligned} E_{kin,rel} &= (\gamma - 1) m_0 c^2 \\ E_{kin,nrel} &= \frac{1}{2} m_0 v^2 = \frac{1}{2} m_0 (\beta \cdot c)^2 \end{aligned}$$

$$\gamma^2 = \frac{1}{1-\beta^2} \rightarrow 1 - \beta^2 = \frac{1}{\gamma^2} \rightarrow \beta^2 = 1 - \frac{1}{\gamma^2}$$

$$\begin{aligned} 1 + 0(\beta) &= \frac{E_{kin,rel}}{E_{kin,nrel}} = \frac{(\gamma-1) m_0 c^2}{\frac{1}{2} m_0 \beta^2 c^2} = \frac{2(\gamma-1)}{\beta^2} \\ &= \frac{2(\gamma-1)}{1-\frac{1}{\gamma^2}} = \frac{2\gamma^2(\gamma-1)}{(\gamma^2-1)} = \frac{2\gamma^2}{\gamma+1} \end{aligned}$$

$$\begin{aligned} \rightarrow 0(\beta) &= \frac{2\gamma^2}{\gamma+1} - 1 = \frac{2\gamma^2 - \gamma - 1}{\gamma+1} = a \\ 2\gamma^2 - \gamma - 1 &= a(\gamma+1) \\ \rightarrow 2\gamma^2 + (-1-a)\gamma - 1 - a &= 0 \\ \gamma^2 - \frac{1+a}{2}\gamma - \frac{1+a}{2} &= 0 \\ \gamma &= \frac{1+a}{4} \pm \sqrt{\left(\frac{1+a}{4}\right)^2 + \left(\frac{1+a}{2}\right)} \end{aligned}$$

Da der Fehler 1% betragen soll, gilt: $a = 0.01$

$$\gamma = \frac{1.01}{4} + \sqrt{\left(\frac{1.01}{4}\right)^2 + \left(\frac{1.01}{2}\right)} = 1.0067$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{1.0067^2}} = 0.115 \rightarrow 11.5\%\, c$$

2 Oberflächenwiderstand und Skintiefe

$$\begin{aligned}\delta &= \sqrt{\frac{2}{\mu_0 \cdot \omega \cdot \sigma}} \\ &= \sqrt{\frac{2}{4\pi \times 10^{-7} \cdot 2\pi \cdot 325 \times 10^6 \cdot 5.8 \times 10^7}} \\ &= 3.67 \times 10^{-6}\, m = 3.67\, \mu m\end{aligned}$$

$$\begin{aligned}R_{ob} &= \frac{1}{\sigma \cdot \delta} \\ &= \frac{1}{5.8 \times 10^7 \cdot 3.67 \times 10^{-6}\, m} \\ &= 4.7\, m\Omega\end{aligned}$$

3 Änderung der Twissparameter beim Linsen-Durchgang

$$A = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \rightarrow c = 1; s = 0; c' = -\frac{1}{f}; s' = 1$$

$$\begin{aligned}\rightarrow \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} &= \begin{pmatrix} 1 & -2 \cdot 0 \cdot 1 & 0^2 \\ \frac{1}{f} & 1 \cdot 1 + 0 & 0 \\ \frac{1}{f^2} & +2 \cdot \frac{1}{f} & 1^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{f} & 1 & 0 \\ \frac{1}{f^2} & +\frac{2}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\rightarrow \beta &= \beta_0 \\ \alpha &= \frac{\beta_0}{f} + \alpha_0 = 0 \\ \rightarrow f &= \frac{\beta_0}{(\alpha - \alpha_0)} = -\frac{\beta}{\alpha_0}\end{aligned}$$

4 F0D0-Struktur

Einzelzelle eines AG-fokussierenden Beschleunigers

$$\begin{aligned}Q_1 &= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \\ A &= \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \\ Q_2 &= \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}\end{aligned}$$

$$M = Q_1 \cdot A \cdot Q_2 \cdot A \cdot Q_1$$

$$
\begin{aligned}
M &= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 - \frac{d}{2f} & d \\ -\frac{1}{2f} & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 - \frac{d}{2f} & d \\ \frac{1}{f} - \frac{d}{2f^2} - \frac{1}{2f} & \frac{d}{f} + 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 - \frac{d}{2f} & d \\ \frac{1}{2f} - \frac{d}{2f^2} & 1 + \frac{d}{f} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 - \frac{d}{2f} + \frac{d}{2f} - \frac{d^2}{2f^2} & d + d + \frac{d^2}{f} \\ \frac{1}{2f} - \frac{d}{2f^2} & 1 + \frac{d}{f} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 - \frac{d^2}{2f^2} - \frac{d^2}{2f^2} & 2d + \frac{d^2}{f} \\ \frac{1}{2f} - \frac{d}{2f^2} & 1 + \frac{d}{f} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 - \frac{d^2}{2f^2} - \frac{d^2}{2f^2} & 2d + \frac{d^2}{f} \\ \frac{1}{2f}\left(1 - \frac{d}{f}\right) & 1 + \frac{d}{f} \end{pmatrix} \\
&= \begin{pmatrix} 1 - \frac{d^2}{2f^2} & 2d + \frac{d^2}{f} \\ -\frac{1}{2f} + \frac{d^2}{4f^3} + \frac{1}{2f}\left(1 - \frac{d}{f}\right) & -\frac{2d}{2f} - \frac{d^2}{2f^2} + 1 + \frac{d}{f} \end{pmatrix} \\
&= \begin{pmatrix} 1 - \frac{d^2}{2f^2} & 2d + \frac{d^2}{f} \\ \frac{d^2}{4f^3} - \frac{d}{2f^2} & 1 - \frac{d^2}{2f^2} \end{pmatrix}
\end{aligned}
$$

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\begin{aligned}
\cos\mu + \alpha \cdot \sin\mu &= 1 - \frac{d^2}{2f^2} \\
\beta \cdot \sin\mu &= 2d + \frac{d^2}{f} \\
-\gamma \sin\mu &= \frac{d^2}{4f^3} - \frac{d}{2f^2} \\
\cos\mu - \alpha \sin\mu &= 1 - \frac{d^2}{2f^2}
\end{aligned}
$$

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\begin{aligned}
\cos\mu + \alpha\sin\mu - \cos\mu + \alpha\sin\mu = 1 - \frac{d^2}{2f^2} - 1 + \frac{d^2}{2f^2} &= 0 \\
2\alpha\sin\mu &= 0 \to \mu = \frac{\pi}{2}
\end{aligned}
$$